This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

ANALYTICAL ARITHMETIC.

BANDY

Educ T 118.90.190

Harvard College Library

FROM

James M. Bandy.

17 Dec. 1891.

Presented by the Author
to Harvard College Library

3 2044 097 001 739

Trinity College Publication, No. IV.

AN

ANALYTICAL ARITHMETIC,

IN SIX PARTS.

BY

James M.,
J. M. BANDY, A. B., A. M.,

Professor of Mathematics and Engineering in Trinity College, N. C.

TRINITY COLLEGE, N. C.:
TRINITY PUBLICATION SOCIETY.
1890.

No. IV.

Educ T 118.90.190
~~Math 4562~~

HARVARD COLLEGE
LIBRARY.

The Author

Entered according to act of Congress, in the year 1890,
By J. M. BANDY,
in the office of the Librarian of Congress at Washington.

PRINTED BY
THOMAS BROTHERS,
GREENSBORO, N. C.

PREFACE.

By most authors Arithmetic has not been so represented as to secure logical habits of mind, the power of independent thought, and that quickess of apprehension which is so sure to follow a course of Analytical Arithmetic.

From this it is to be inferred that Rules and Mechanical devices by which the above-mentioned good effects are not obtained will be ignored. One new feature of this book, therefore, will be that after donning the Analytical dress, a Rule shall not appear on every page.

To give the student the mastery over business problems of life; and, at the same time, should it be desired, to enable him to successfully study the higher mathematics, are the objects of this book.

It is to be placed in the hands of those who can add, substract, multiply, and divide.

SUGGESTIONS.

This book being Analytical throughout does not propose to sacrifice Reasoning for time and brevity. Were it to do so, it would fail to accomplish its object. Time and Patience must be exercised by both teacher and pupil. If it take time to go through with all the Analysis, it is its highest recommendation. Rome was not built in a day.

The reasoning Faculties are not developed without exercise. Take the time to drill.

AN EXPLANATION

OF SIGNS USED IN THIS BOOK.

$\therefore$ is to be read "*therefore.*"

$\because$ is to be read "*since.*"

×, standing between numbers, is to be read "*so many times*" the number following it.

= is to be read "*equals.*"

(N. B.—This last character is not used in the same sense in which it is used in algebra.)

1. **Arithmetic** is naturally divided into two parts: Whole Numbers and Fractions. **Whole Numbers** will be treated first.

2. A **Number is a Unit,** or a **Collection of Units**; as **One Apple, Twelve Apples.**

3. A **Unit** is a single thing; as One Marble. It is the basis of all Numbers, and, therefore, the basis of that knowledge which leads to an intelligent understanding of Arithmetic.

4. Analysis in Arithmetic makes the Unit the Basis of all Reasoning. In reasoning, therefore, if we do not start with the Unit, we must pass from the Given Number (=Collection of Units) to its Unit; then, to the Number of required Units.

5. It is claimed that the **Analyses,** which appear in this book, possesses a uniformity not found in any other book. They are simpler than those found in any other book, and, therefore, clearer.

PART I.

WHOLE NUMBERS.

CHAPTER I.

1. If one bushel of wheat cost 80 cents, what must be paid for 20 bushels?

6. Analysis.

1 bushel=80 cents,
20 bushels=20×80 cents=$16.00.

2. Bought 45 cords of wood at 4 dollars a cord; what was the cost of the wood?

3. A drover bought 164 head of cattle at 8 dollars a head; what did he pay for all?

4. If a piece of calico contains 27 yards; how many yards are contained in 9 pieces?

5. If a steam boat goes at the rate of 14 miles per hour; how far will it go in 6 hours?

6. If 16 yards of cloth will cover an office floor; how many yards will be required for 7 offices?.

---

*This mark *must* be read "times"; thus, in the above *twenty times* 80 cents. Not *multiplied by*. This *manner* of reading *must be strictly observed.*

7. If 6 men can do a piece of work in 3 days; how long will it require 1 man to do it?

7. Analysis:

∵ to do the work of 6 men = 3 days*,

" " 1 man = 6 × 3 days = 18 days.

8. If 7 men can build a barn in 8 days; how long will it take 1 man to do it?

9. If 6 men can do a piece of work in 9 days; how long will it take 1 man to do it?

10. If 4 men can build a wall in 12 days; how many men can build it in 1 day?

Note.—Observe that the required element here is men. Let it stand last thus:

8. Analysis:

∵ to build the wall in 12 days = 4 men,

" " 1 day = 12 × 4 men = 48 men.

11. If 8 men can earn 25 dollars in 6 days; how many men can earn the same in 1 day?

12. If a barrel of flour last a family of 6 persons 8 weeks; how long will it last a family of one person?

13. How many men can do in 9 days what 18 men can do in 1 day?

14. How many men can earn the same money in 11 days that 77 men can earn in 1 day?

15. If 30 loaves of bread would supply 27 persons for 1 day; how many persons would they supply for 9 days?

---

*Always let the *required element stand last, after the equal marks*.

9. Analysis:

∵ 30 loaves of bread in 1 day=27 persons,
30 " " 9 days=$\frac{1}{9}$ of 27 persons=$\frac{27}{9}$=3 persons.

16. If 2 men can construct an engine in 15 months; how long will it take 6 men to construct it?

17. If 9 men can do a piece of work in 16 days; how long will it take 12 men to do it?

18. If 9 men can do a piece of work in 16 days; how many men will do it in 12 days?

19. How long would it take 12 men to do what 8 men would do in 6 days?

20. If $100 will support a family of 7 persons for 8 weeks; how many persons would it support for 4 weeks?

21. A ship's crew of 600 men have provisions sufficient to allow each man 9 ouncss a day; if one-third be withdrawn, how many ounces a day will be allowed the remainder?

10. Analysis:

∵ 1 man=9 ounces,
600 men=600×9 ounces=5400 ounces.

Now ∵ $\frac{1}{3}$ of the men is withdrawn, there remains $\frac{3}{3}$—$\frac{1}{3}$=$\frac{2}{3}$.

But ∵ $\frac{3}{3}$=600 men,
$\frac{1}{3}$=$\frac{1}{3}$ of 600 men=200 men,
$\frac{2}{3}$=2×200 men=400 men.

∴ ∵ 400 men in 1 day=5400 ounces,
1 man in 1 day=$\frac{1}{400}$ of 5400 ounces=$\frac{5400}{1}$13$\frac{1}{2}$ ozs.

22. If 7 yards of broad cloth be worth 28 dollars; what will 13 yards be worth?

23. If 6 hats cost 24 dollars; what will 110 hats cost?

24. If a certain piece of grass can be mown in 6 days by 20 men, and one-half of them be called away, in what time will the remainder complete the work?

LESSON II.

I. A farmer sold 6 barrels of flour at 5 dollars a barrel, and took his pay in cloth at 3 dollars a yard; how many yards of cloth should he receive?

II. Analysis:

∵ 1 barrel of flour=5 dollars,
6 barrels " =6×5 dollars=30 dollars.
∵ 3 dollars=1 yard of cloth,
1 dollar=$\frac{1}{3}$ of a yard.
and 30 dollars=30× $\frac{1}{3}$ of a yard=$\frac{30}{3}$=10 yards.

2. How much honey, at 12 cents a pound, must be given for 6 pounds of coffee, at 16 cents a pound?

3. How many hats, at 3 dollars a piece, must be given for 15 yards of cloth at 4 dollars a piece?

4. How many oranges, at 3 cents a piece, must be given for 18 lemons at 2 cents a piece?

5. A farmer has 5 dozen eggs, worth 10 cents a dozen, and wishes to exchange them for nutmegs worth 2 cents a piece; how many nutmegs should he receive?

6. A merchant gave 12 boxes of lemons for two firkins of butter, worth 3 dollars a firkin; what was the price of the lemons a box?

7. An employer owed his servant 4 month's wages, at 10 dollars a month; he agreed to pay him 12 dollars in cash, and the remainder in flour at 3 dollars a barrel; how many barrels did he get?

8. How many chickens, at 3 shillings a piece, must be given for 12 turkeys at 9 shillings a piece?

9. A farmer agreed to take 9 yards of cloth, at 3 dollars a yard, in payment for 9 calves; what did he receive a piece for his calves?

10. How many chickens, at 8 cents a piece, will pay for 10 yards of calico, at 8 cents a yard?

LESSON III.

1. What is the cost of 50 lemons, at 2 for 4 cents?

12 Analysis:

$\because$ 2 lemons $=$ 4 cents,

1 lemon $= \frac{1}{2}$ of 4 cents $= \frac{4}{2} =$ 2 cents.

and 50 lemons $= 50 \times 2$ cents $=$ \$1.00.

2. What is the cost of 36 oranges at the rate of 3 for 5 cents?

Note.—Observe to place what is required always after the equal marks, last, beginning the analysis with the ***known unrequired*** element. There are two elements here mentioned, lemons and money. Money is the required element and lemons the unrequired element. 2 lemons is the known unrequired element, because its value is known. The advantage of cancellation should be taken. Because we analyze, we should not fail to cancel terms when we can. In the above, analysis will arrange the matter thus:

13. Analysis:

∵ 3 oranges $=5$ cents,

1 orange $=\frac{1}{3}$ of 5 cents $=\frac{5}{3}$ cents,

and 36 oranges $=36\times\frac{5}{3}$ cents $=\frac{36\times5}{3}$* $=60$ cents.

3. What is the cost of 45 apples, at the rate of 3 for 7 cents?

4. How many melons, at 8 cents each, must be given for 60 pencils, at the rate of 3 for 4 cents?

5. If 8 melons cost 14 cents, what will 40 melons cost?

6. If 9 pineapples will buy 36 peaches, and 4 peaches will buy 1 cocoanut, how many cocoanuts will 3 pineapples buy?

7. What is the cost of 48 chickens, if 10 chickens cost 80 cents?

8. How many dozen apples, at 9 cents a dozen, must be given for 27 lemons, at the rate of 15 for 45 cents?

---

*Here 3 into 36=12, 12 multiplied by 5=60.

9. If 8 pencils are worth 12 cents, how many marbles, at 4 cents a piece, can be bought with 72 pencils?

10. If 6 pounds of lard are worth 13 cents, how many yards of muslin, at 2 cents a yard, can be bought for 24 pounds of lard?

LESSON IV.

1. How many cents more will be necessary to give an equal amount to 8 others, if a father divides 30 cents equally among 6 children?

2. How far will a steamboat run in 16 hours, if it runs 50 miles in 5 hours?

3. If 9 calves cost 54 dollars, what will 7 calves cost?

4. If a dog chasing a fox, runs 27 miles in 3 hours; how far will he run in 6 hours?

5. How many pineapples, at 9 cents a piece, must be given for 15 peaches, at 3 for 9 cents?

6. How many yards of cloth, 4 dollars a yard, must be given for 12 barrels of flour at the rate of two barrels for 14 dollars?

7. John had 9 oranges, worth 4 cents a piece, and James 2 pineapples, worth 10 cents a piece; James gave his pineapples for their value in oranges: how many oranges had James left?

8. Two persons are 96 miles apart, and traveling to-

wards each other, one at the rate of 5 miles an hour and other at the rate of 3 miles an hour; how many hours before they will be together?

9. A telegraphic wire is stretched across a bay 30 miles wide, and one of the vessels sails at the rate of 2 miles an hour, and the other in an opposite direction at the rate of 3 miles an hour; how long will they be dropping the wire?

10. A fox is 70 rods ahead of a grayhound, but the grayhound runs 30 rods a minute, while the fox rung 16 rods; in how many minutes will the hound overtake the fox?

11. A saw mill has two steam engines; one of them requires 3 tons of coal per month and the other requires 4 tons per month: how long would a cargo of 168 tons run the engines?

12. John and Henry are on opposite sides of a wood, 450 yards in circumference, when they begin to travel in the same direction, the former at the rate of 25 yards a minute and the latter at the rate of 20 yards; how long will it require John to overtake Henry?

13. A hare is 300 of its leaps ahead of a dog. The hare makes 5 leaps while the dog makes 4; but 2 dog leaps are equal to 3 hare leaps. How many leaps must the dog take to overtake the hare?

14. A captain reached a bridge in time to see the enemy safe on the other side, who retreated at the rate of 16 miles a day. It took 4 days to repair the bridge. The captain then pursued at the rate of 24 miles a day; how many days before he overtook the enemy?

15. How many lines of partition will be required in a stable that is 75 feet deep, so as to allow 3 feet for each space?

16. In a lumber mill 5 men and 4 boys are employed, and the weekly wages of them all amounts to 70 dollars. The wages of one man is equal to that of two boys. What does each boy and each man get?

17. John and George can do a piece of work in 10 days. John can do twice as much in a day as George. After working together 6 days, George is left to finish it; how long will it take him?

18. If 3 men and 4 boys earn 50 dollars in 10 days, when a boy gets half as much as a man, what does each man and each woman get in one day?

19. 9 colts and 6 calves are pastured for 4 weeks for 44 dollars, a colt costing 3 times as much as a calf. What is paid for each colt and each calf?

CHAPTER II.

LESSON I.

14. It is proposed here to treat by analysis such problems as have been formerly classed under the **Double Rule of Three.** As there are only three classes of problems that come under this head, each class, together with its analysis, will be considered in a separate lesson.

1. If 4 men in 3 days can chop 36 cords of wood, how many cords can 5 men chop in 7 days?

15. Men, animals, machinery, &c., acting in time is called **Cause.** What they do, or accomplish, is called **Effect.** In the above problem, **Effect** is wanted, and the analysis is as follows:

16. Analysis:

$\because$ 4 men in 3 days $=36$ cords of wood,
1 man in 3 days $=\frac{1}{4}$ of 36 cords $=\frac{36}{4}=9$ cords,
$\therefore$ 1 " 1 day $=\frac{1}{3}$ of 9 " $=\frac{9}{3}=3$ "
5 men in 1 " $=5\times$*3 " $=15$ "
and 5 " 7 days $=7\times15$ " $=105$ cords.

Note.—For the purpose of developing analysis under this head, I shall set this paper with some problems found in other books under the subject of Interest. Besides showing the analysis, problems in interest are as practical as any.

---

*Must be careful to read *five times* 3 cords. Also, place cords after the equal marks because cords is required.

2. If 20 dollars in 3 years produce $4.80, what will 7 dollars produce in 5 years?

3. Paid 60 dollars for a horse; kept him 15 days and sold him for 70 dollars. At this rate, what would 20 dollars in 10 days produce?

4. If 100 dollars will board 4 persons for 5 weeks, how much will be required to board 12 persons for 15 weeks?

5. If 7 horses in 4 days eat 28 acres of grass, how many acres will subsist 9 horses for 12 days?

6. If 5 steam engines in 4 months consume 100 tons of coal, how much coal will 3 engines consume in 7 months?

7. If 3 wheat drills, with 7 points each, in 4 days can sow 48 acres; how many acres can 5 drills, with 9 points each, in 6 days sow?

Note.—3 drills with 7 points each = 1 drill with 21 points.

8. A man bought 25 bushels of oats at 50 cents a bushel, and carried them to market, which took 8 days; sold them for 95 cents a bushel. What did he receive on a dollar in a year of 360 days?

9. If 7 bushels of wheat are worth 14 bushels of rye, and 5 bushels of rye are worth 10 bushels of oats, and 6 bushels of oats are worth 3 dollars; how many bushels of wheat will 30 dollars buy?

10. If an army of 1200 men have provisions for 20 days, at 10 ozs. a day to each man, and they be reinforced by 600 men, upon what allowance per day must each man be put, that the same provisions may last 30 days?

LESSON II.

1. If 4 men in 3 days chop 36 cords of wood, how many men in 7 days can chop 105 cords?

Note.—Observe that this is the first problem in previous lesson, changed. In the previous lesson **Cords, Effect,** was required; but in the problem, as it now is, **Men,** an **Element** of **Cause,** is required.

17. Analysis:

∵ to chop 36 cords of wood in 3 days=4 men,
" 1 cord " 3 " $=\frac{1}{36}$ of 4 men $=\frac{4}{36}$
$=\frac{1}{9}$ of a man,
" 1 " " 1 day $=3\times\frac{1}{9}$ men $=\frac{1}{3}$ men,
" 105 cords " 1 " $=105\times\frac{1}{3}$ men
$=35$ men,
and " " " " 7 days $=\frac{1}{7}$ of 35 men $=\frac{35}{7}$
$=5$ men.

Note.—Let element men stand last, because it is required.

2. If 5 men in 6 days can excavate 60 cubic feet of earth, how many men in 8 days can excavate 160 cubic feet?

3. If 20 men in 12 days can print 700 copies of a book, how many men in 8 days can print 350 copies?

4. If 12 dollars in 20 days gain 4 dollars; how many dollars in 30 days will gain 5 dollars?

5. If 7 men in 14 days can mow 84 acres of grass, how many men in 4 days can mow 72 acres?

6. If an army of 1200 men have provisions for 20 days, at 10 ozs. a day for each man; by how many men must the army be increased that the provisions, at $4\frac{4}{9}$ ozs. a day to each man, may last 30 days.

7. If four cars in 5 days transport 40 tons of coal; how many cars is 6 days will transport 108 tons?

LESSON III.

1. If 4 men in 3 days can chop 36 cords of wood; in how many days can 5 men chop 105 cords of wood?

Note.—Observe again that this is the first problem in lesson 1 of this chapter. The change now required is the *element* days. It should, therefore, stand last in the analysis, thus:

$\because$ to chop 36 cords of wood by 4 men $=3$ days,
" 1 cord " 4 " $=\frac{1}{36}$ of 3 days $=\frac{3}{36}$
$=\frac{1}{12}$ of a day,
" 1 " " 1 man $=4\times\frac{1}{12}$ of a day $=\frac{4}{12}$
$=\frac{1}{3}$ of a day,
" 105 cords " 1 " $=105\times\frac{1}{3}$ of a day
$=\frac{105}{3}=35$ days,
and " " " " 5 men $=\frac{1}{5}$ of 35 days $=\frac{35}{5}$
7=days.

Note.—The pupil should compare the three last analyses carefully. He should observe the logical force which makes 35 in the last example of the *quality* of *days*, and in the second lesson of the *quality* of *men*.

2. If 8 men in 6 days dig 72 cubic feet of earth; in how many days will 10 men dig 135 cubic feet?

3. If 20 dollars in 30 days gain 12 dollars; in how many days will 100 dollars gain 28 dollars?

4. If 6 wheat drills, with 6 points each, in 4 days can set 120 acres in wheat; in how many days can 6 drills, with 9 points each, set 225 acres in wheat?

5. If 2 men, in 5 days, can build 160 feet of wall; how long will it take 4 men to build 200 feet?

6. A person having a journey of 500 miles to perform, walks 200 milles in 8 days; in how many days will he complete the journey?

7. If 60 bushels of corn, in 40 days, will feed 24 horses; how long will 30 bushels feed 48 horses?

8. If 5 men, in 3 days, chop 50 cords of wood; in how many days will 4 men chop 64 cords?

9. If in 5 days 12 horses plow 11 acres; in how many days will 10 horses plow 33 acres.

These three lessons show the different forms in which problems of this kind can arise. The analysis of each case should be carefully comprehended.

LESSON IV.

1. If 16 men in 4 days cut 128 cubic yards of earth; how much can 12 men in 8 days cut?

2. If 16 men in 4 days cut 128 cubic yards of earth; how many men in 8 days can cut 192 yards of earth?

3. If 16 men in 4 days cut 128 cubic yards of earth; in how many days can 12 men cut 192 yards?

4. If 500 men have provisions in meat for 30 days at 9 oz. a day; what must be their daily allowance, if reinforced by 400 men, and the time is extended 15 days?

5. If 200 dollars in 30 days gain 12 dollars; what will 300 dollars gain in 18 days?

6. If a 5-cent loaf weighs 7 oz., when flour is 5 dollars a barrel; how much should it weigh when flour is worth 6 dollars a barrel?

7. If a stream of water running into a pond of 120 acres, raises it 12 inches in 16 hours; how much would a pond of 60 acres be raised by the same stream in 12 hours?

8. If 4 printers in 30 days, 10 hours each, can print 300 copies of a book; how many printers in 20 days, 8 hours each, can print 200 copies?

9. If 200 dollars in 8 months gain 16 dollars; in what time would 500 dollars gain 15 dollars?

10. If 500 dollars in 3 months gain 15 dollars; how many dollars in 8 months will gain 16 dollars?

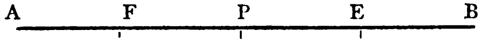
PART II.

COMMON FRACTIONS.

CHAPTER I.

18. A **Fraction** is a part of a **Unit,** or a part of any **Quantity** considered as a **Whole.**

19. Thus let



A B be **Any Length** of a stick, and let it be divided at the point P into **Two Equal Parts. Each Part** is **one-half** of the **Whole Stick,** and is written thus: $\frac{1}{2}$.

Again, let A B be divided at the points E, P, F, into **Four Equal Parts. Each Part** is **One-Fourth** of the **Whole Stick,** written thus: $\frac{1}{4}$. A E is **Three-Fourths** of the **Whole Stick**; thus: $\frac{3}{4}$.

Note.—It is insisted upon that the teacher should provide himself with an apple, cut it into halves, fourths, &c., showing the pupil that a fraction is a part of some whole thing.

20. The **Manner** in which the **Unit** is divided determines the **Kind** of **Fraction.**

21. The **Unit** may be divided into **Any Number** of **Equal Parts**; as halves, fourths, fifths, &c. Hence, **Common Fractions.**

22. A **Common Fraction,** therefore, is one whose **Unit** is divided into **Any Number** of equal parts.

23. **Every Fraction** consists of **Two Terms**; one term written above a **Horizontal** line, the other **Term** written below a **Horizontal** line; thus $\frac{3}{4}$.

24. The **Term** written above the horizontal line is called the **Numerator.**

25. The **Term** written below the horizontal line is called the **Denominator.**

Let the pupil distinguish carefully between the **Unit** of the fraction and the **Fractional Unit.**

26. In the fraction $\frac{3}{4}$, $\frac{4}{4}$ is the **Unit** of the fraction, and $\frac{1}{4}$ is the **Fractional Unit.**

27. A **Proper Fraction** is a fraction whose **Numerator** is **Less** than its **Denominator**; as, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{13}{14}$.

28. An **Improper Fraction** is a fraction whose **Numerator** is **Equal** to, or **Exceeds,** the **Denominator,** as, $\frac{5}{5}$, $\frac{8}{3}$, $\frac{15}{13}$.

29. A **Mixed Number** consists of a **Whole Number** and a **Fraction**; thus: $2\frac{4}{5}$, $7\frac{3}{10}$, $8\frac{5}{6}$.

30. **Reduction** of **Fractions** is changing their **Form** without **Altering** their **Value.**

31. To **Reduce** a fraction to its **Lowest** terms is to divide both the numerator and denominator by any number that will divide each one of them without a **Remainder.** When this operation reaches **Terms** both of which can not

be divided without a remainder, the fraction is reduced to its **Lowest** terms.

32. Thus: reduce $\frac{12}{16}$ to its lowest terms. 4 is the number that will divide both terms of the fraction $\frac{12}{16}$, *without* a *remainder*, reaching, as a result, a fraction $\frac{3}{4}$. Here no number, except 1, will divide both terms of the fraction $\frac{3}{4}$, without a remainder.

∴ by cond. (31), $\frac{3}{4}$ is the lowest terms of the fraction $\frac{12}{16}$.

Since the **Lowest** terms of a fraction are the terms sought, it is obvious that the pupil must seek the **Greatest** divisor common to both terms of the fraction. 4 in (32) is the **Greatest Common Divisor** of 12 and 16, the terms of the fraction $\frac{12}{16}$.

33. The **Greatest Common Divisor** (=G. C. D.) of two, *or more*, numbers is the greatest number that will divide them without a **Remainder.**

Note.—**Observe that the terms of a fraction are whole numbers.**

The property of the G. C. D. of numbers being necessary to a treatment of the subject of fractions, we will observe that the G. C. D. of two, or more, numbers may be obtained by **Inspection,** by **Prime Factors,** or by a **Process** of division.

34. By **Inspection,** find the G. C. D. of (6, 8), (6, 8, 12) (12, 14) (4, 6, 8, 10), (9, 6, 22). Considering the numbers that would divide 6 and 8, we observe that 2 is the only

number that will divide both without a remainder; and it is also the **Greatest** number that will divide them without a remainder.

$\therefore$ G. C. D. of 6, 8=2.

Consider the other groups in the same way.

35. By **Prime Factors.** But we must first explain what are **Prime Factors.** Let us examine the number 24 by the law of multiplication. We see that

$$24=4\times6, \quad (1)$$
$$=8\times3, \quad (2)$$
or
$$=2\times12. \quad (3)$$

In (1) 4 and 6 are **Factors,** of 24, and in (2) 8 and 3 are **Factors** of 24, and in (3) 2 and and 12 are **Factors** of 24.

36. A **Factor** of a number, therefore, is a number which multiplied by another number produces the number.

Let us take 24 again.

$$24=2\times2\times2\times3.$$

Here 2, 2, 2, 3 are **Prime Factors** of 24, They are called **Prime Factors,** because no number, except itself and 1, will divide each without a remainder.

37. A **Prime Factor,** therefore, is one that cannot be divided by any other number, except itself and 1, without a remainder.

38. By prime factors, find the G. C. D. of 12,18, 24.

Prime factors of $12=2\times2\times3$,
" " $18=2\times3\times3$,
and " " $24=2\times2\times2\times3$.

Here the prime factors are *twos* and *threes*. In the numbers 12 and 24, 3 enters *once*. This, therefore, is the *highest* number of times it can enter as a common divisor. Hence, take 3 as a factor *once*. Again, we observe that 2 enters 18 *once*. Hence, take 2 once.

∴ G. C. D. of 12, 18, $24=2\times3=6$.

Again find the G. C. D. of 24, 36, 64.

The prime factors of $24=2\times2\times2\times3$,
" " " $36=2\times2\times3\times3$,
" " " $64=2\times2\times2\times2\times2\times2$.

Here in 64 3 enters *not*. It can not enter as a common divisor at all. In 36, 2 enters *twice*. It cannot enter as a common divisor *more* times.

∴ G. C. D. of 24, 36, $64=2\times2=4$.

∴ the G. C. D. of two, or more, numbers is the product of all the common prime factors, each prime factor taken as many times as the *least* number of times by which it has entered one of the numbers.

Note. The teacher should give a thorough drill on this by furnishing other examples. This method is used when the numbers are larger than those numbers where G. C. D. can be obtained by inspection.

39. When the numbers are very large, to find G. C. D. by division. Let us seek the G. C. D. of 98 and 140.

Dividing the greater by the less,

$\frac{140}{98}=1+$remainder 42. (1.)

Dividing 98 by 42,

$\frac{98}{42}=2+$remainder 14. (2.)

Dividing 42 by 14,

$\frac{42}{14}=3+$remainder 0. (3.)

Here the **Process** was to divide the **Larger** number by the **Smaller** number; which left a remainder 42. [See (1.)]. Then (2) to divide 98, the previous divisor, by the remainder 42; which left 14 for remainder. Then (3) to divide 14, the previous divisor, by 14, which left nothing for remainder.

The **Process,** therefore is to divide the **Larger** number by the **Smaller** number; then divide the **Previous** divisor by the **Least** remainder till a Remainder, which divides the last divisor without a mainder, is reached. This last divisor is the G. C. D. of the numbers.

40. To prove that 14 is:

I. A common divisor of 98 and 140.
II. The G. C. D.

Principle I. Any number that will divide two numbers, each taken separately without a remainder, will divide their **Sum.** For example,

$\frac{12}{6}=2,$ (1.)
$\frac{18}{6}=3,$ (2.)
But $\frac{12+18}{6}=\frac{30}{6}=5;$ [their sum.] (3.)

Principle II. Any numbber that will divide two numbers, each taken separately, without a remainder, will divide their **Difference.** For example,

$\frac{12}{6}=2,$ (1.)
$\frac{18}{6}=3,$ (2.)
But $\frac{18-12}{6}=\frac{6}{6}=1;$ [their difference.] (3.)

Principle III. Any number that will divide a number without a remainder, will divide the *same* number multiplied by *any number.* For example,

$\frac{12}{6}=2,$ (1.)
$\frac{7\times12}{6}=\frac{84}{6}=14;$ [7 being any number]. (2.)

Now 42 is the difference between 140 and 98; [39(1)]. But 14 *Divides* 42; [39(3)].

∴ 14 is a **Common** divisor of 98, and 140, by [40, Prin. II., (3)]. Hence, proof for why 14 is their **Common** divisor.

Again, 14 divides 42×2; [40, Prin. III. (2)].

But $42 \times 2 = 84$.

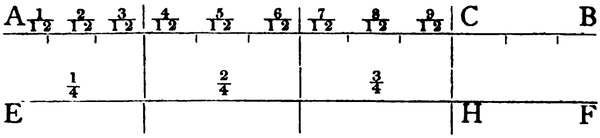
But $\frac{84+14}{14}$; 40 Prin. 1.]

But 14 is the highest number that will divide 14.

∴ 14 is their G. C. D.

LESSON I.

41. To reduce fractions to their lowest terms by **Inspection.**



Let A B and E F be two lines of the **Same Length;** and let A B be divided into twelve **Equal** parts and E F into four **Equal** parts. Let these lines be so placed that the end A is even with the end E, and the end B is even with the end F. Then, if A B is cut in two at C and E F is cut in two at H,

$$A\,C = E\,H. \qquad (1)$$

But counting the **Equal** parts from A towards B, we find

$$A\,C = \tfrac{9}{12};$$

and counting the **Equal** parts from E towards F, we find

$$E\,H = \tfrac{3}{4}$$

But in (1) $A\,C = E\,H.$

$$\therefore \quad \tfrac{9}{12} = \tfrac{3}{4}. \qquad (2)$$

But $\tfrac{9}{12} = \frac{9 \div 3^{*}}{12 \div 3} = \tfrac{3}{4}. \qquad (3)$

$\therefore \because$ in (2) $\tfrac{9}{12} = \tfrac{3}{4}$, if we divide the *numerator* and the *denominator*, both terms of a fraction, by the **Same** number, the **Value** of the fraction will not be changed.

Note.—Observe that 3 in (3) is the G. C. D. of 9 and 12, the terms of the fraction $\tfrac{9}{12}$.

$^{*}\frac{9 \div 3}{12 \div 3}$ is to be read "$\tfrac{9}{12}$ each term divided by 3."

Arranging the work as in (3) above, reduce the fractions to their lowest terms:

1. $\frac{15}{20}$
2. $\frac{12}{16}$
3. $\frac{6}{8}$
4. $\frac{10}{12}$
5. $\frac{12}{14}$
6. $\frac{9}{12}$
7. $\frac{8}{10}$
8. $\frac{7}{14}$
9. $\frac{12}{15}$
10. $\frac{15}{25}$
11. $\frac{6}{10}$
12. $\frac{12}{18}$
13. $\frac{15}{28}$
14. $\frac{18}{30}$
15. $\frac{24}{30}$
16. $\frac{7}{21}$
17. $\frac{12}{24}$
18. $\frac{8}{24}$

LESSON II.

42. To reduce fractions to their lowest terms by **Prime Factors.** Let us take $\frac{36}{64}$ for the purpose of showing the *arrangement* of the work.

Thus: prime factors of $36=2\times2\times3\times3$,*

and " " " $64=2\times2\times2\times2\times2\times2$.

$\therefore$ by (39) G. C. D. of 36 and $64=2\times2$; [the least number of times 2 has entered].

$$\therefore \qquad \frac{36}{64}=\frac{36\div4}{64\div4}=\frac{9}{16}$$

In the same way, reduce the following fractions to their lowest terms:

1. $\frac{42}{96}$
2. $\frac{48}{144}$
3. $\frac{26}{104}$
4. $\frac{24}{72}$
5. $\frac{45}{100}$
6. $\frac{34}{106}$
7. $\frac{72}{84}$
8. $\frac{25}{200}$
9. $\frac{36}{70}$
10. $\frac{28}{84}$
11. $\frac{125}{275}$
12. $\frac{81}{109}$
13. $\frac{44}{84}$
14. $\frac{144}{240}$
15. $\frac{84}{252}$

*Observe that the prime factor 3 is not common to both. It has not entered 64 at all.

LESSON III

53. To reduce a fraction to its lowest terms by (40) **Division.** For the purpose of showing the arrangement of the work let us take $\frac{294}{378}$

Arrange thus:

$\frac{378}{294}=1+$ remainder 84.

$\frac{294}{84}=3+$ " 42.

$\frac{84}{42}=2+$ " 0.

∴ by (40) $\frac{378}{294}=\frac{378\div 42}{294\div 42}=\frac{7}{9}$

In the same way, reduce the following fractions to their lowest terms:

1. $\frac{784}{1162}$
2. $\frac{256}{324}$
3. $\frac{1216}{1472}$
4. $\frac{242}{338}$
5. $\frac{896}{1682}$
6. $\frac{278}{896}$
7. $\frac{189}{297}$
8. $\frac{627}{1023}$
9. $\frac{355}{868}$
10. $\frac{272}{764}$
11. $\frac{912}{1516}$
12. $\frac{762}{1492}$
13. $\frac{2345}{9052}$
14. $\frac{154}{352}$
15. $\frac{186}{369}$

Note.—Every fraction should be reduced to its lowest terms.

LESSON IV.

44. To reduce an Improper* fraction to a Whole, or Mixed number. Let us reduce $\frac{26}{4}$ to a whole, or mixed, number.

---

*An improper fraction must be regarded as a collection of so many units and parts, or parts of another unit.

45. Analysis:

∵ $\frac{4}{4}=1$ whole number,

$\frac{1}{4}=\frac{1}{4}$ of 1 W. N.$=\frac{1}{4}$ of W.N.,

and $\frac{25}{4}=25\times\frac{1}{4}$ W. N.$=\frac{25}{4}=6\frac{1}{4}$ W. N.

Let the following be analyzed in the same manner:

1. $\frac{25}{5}$
2. $\frac{17}{3}$
3. $\frac{18}{4}$
4. $\frac{125}{8}$
5. $\frac{16}{4}$
6. $\frac{157}{12}$
7. $\frac{160}{25}$
8. $\frac{29}{3}$
9. $\frac{145}{7}$
10. $\frac{164}{6}$
11. $\frac{144}{12}$
12. $\frac{16}{7}$
13. $\frac{256}{8}$
14. $\frac{300}{5}$
15. $\frac{560}{4}$

46. Having learned the reason, this operation in practical work should be performed thus:

$$\frac{157}{12}=13\frac{1}{12}.$$

In the same manner, reduce the following fractions to whole, or Mixed, numbers:

1. $\frac{17}{2}$
2. $\frac{676}{25}$
3. $\frac{925}{25}$
4. $\frac{144}{12}$
5. $\frac{576}{28}$
6. $\frac{625}{15}$
7. $\frac{802}{4}$
8. $\frac{112}{3}$
9. $\frac{1865}{30}$
10. $\frac{23}{2}$
11. $\frac{84}{7}$
12. $\frac{132}{14}$
13. $\frac{1534}{34}$
14. $\frac{1987}{62}$
15. $\frac{800}{20}$

LESSON V.

46. To reduce a **Whole** number to a fraction having a **Given** denominator. Let us reduce 8 to a fraction whose denominator shall be 15.

47. **Analysis:**

∵ 1 unit$=\frac{15}{15}$,

∵ 8 units$=8\times\frac{15}{15}=\frac{120}{15}$

In the same manner, reduce the following whole numbers to fractions having a G. D.

1. 9 to a fraction with denominator 7,
2. 12 " " " 13,
3. 6 " " " 9,
4. 19 " " " 22,
5. 13 " " " 12,
6. 40 " " " 9.

LESSON VI.

48. To reduce a **Mixed** number to an **Improper** fraction. Let us reduce $13\frac{1}{12}$ to an Improper fraction.

49. Analysis:

$$1 \text{ unit} = \frac{12}{12}{}^{*}$$

$$13 \text{ units} = 13 \quad \frac{12}{12} = \frac{156}{12}.$$

$$13\frac{1}{12} \text{units} = \frac{156}{12} + \frac{1}{12}{}^{\dagger} = \frac{157}{12}$$

Let the following be analyzed in the same manner.

1. $8\frac{3}{4}$
2. $15\frac{1}{6}$
3. $12\frac{1}{2}$
4. $33\frac{1}{3}$
5. $11\frac{1}{5}$
6. $19\frac{1}{4}$
7. $18\frac{3}{4}$
8. $27\frac{2}{3}$
9. $31\frac{4}{5}$
10. $66\frac{2}{3}$
11. $14\frac{7}{8}$
12. $6\frac{1}{4}$
13. $20\frac{6}{7}$
14. $75\frac{3}{4}$
15. $5\frac{2}{17}$

50. Having learned the reason, this operation in practice is performed thus:

$$15\frac{1}{6} = \frac{6 \times 15 + 1}{6} = \frac{91}{6}$$

---

*Begin with $\frac{12}{12}$ because the denominator of the fraction connected with 13 is 12.

† 1 hog can be added to 156 hogs; so can $\frac{1}{12}$ be added to $\frac{156}{12}$ by adding the numerators. The denominators being alike, the numerators can be added.

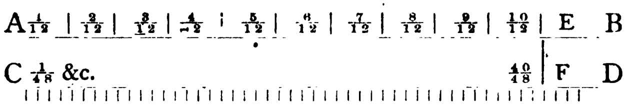
As *neat* and *orderly* arrangement is an instrument of power in communicating mathematical *concepts*, let the following be performed as above:

1.	$17\frac{6}{7}$	6.	$75\frac{9}{25}$	11.	$82\frac{7}{12}$
2.	$144\frac{2}{3}$	7.	$56\frac{5}{9}$	12	$25\frac{4}{13}$
3.	$225\frac{4}{5}$	8.	$63\frac{7}{9}$	13.	$100\frac{2}{3}$
4.	$576\frac{1}{3}$	9.	$52\frac{7}{8}$	14.	$\frac{22}{7}$
5.	$127\frac{1}{7}$	10.	$19\frac{4}{5}$	15.	$16\frac{1}{9}$

LESSON VII.

51. To reduce a fraction to **Higher** terms.

Note.—This is the converse of (41)



Let AB and CD be two lines of the **Same** length; let AB be divided into 12 **Equal** parts, and CD into 48 **Equal** parts. Let these lines be so placed that the end A is even with C, and the end B is even with the end D. Then, if AB be cut in two at E, and CD be cut in two at F,

$$AE = CF. \qquad (1)$$

But counting the **Equal** parts from A towards B, we find

$$AE = \frac{10}{12};$$

and counting the **Equal** parts from C towards D, we find

$$CF=\frac{40}{48}.$$

But from (1) $AE=CF.$

$\therefore$ $\frac{10}{12}=\frac{40}{48}.$ (2)

But $\frac{10}{12}=\frac{10}{12}\times\frac{4}{4}=\frac{40}{48}.$ (3)

$\therefore$ $\because$ in (2) $\frac{10}{12}=\frac{40}{48}$, if we multiply the *numerator* and the *denominator*, both terms of a fraction, by the **Same** number, the **Value** of the fraction will not be changed.

Note.—Observe in (3) that we go from *twelfths* to *forty-eighths*.

The above is *graphical* analysis.

52. Reduce $\frac{10}{12}$ to a fraction whose denominator shall be 48.

Analysis:

$\because$ $\frac{12}{12}=\frac{48}{48}$,

$\frac{1}{12}=\frac{1}{12}$ of $\frac{48}{48}=\frac{4}{48}$,

and $\frac{10}{12}=10\times\frac{4}{48}=\frac{40}{48}.$

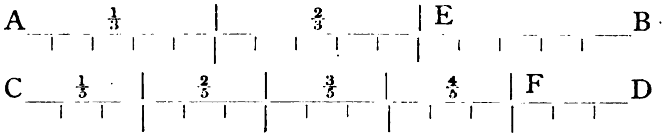
In the same way analyze the following:

1. Reduce $\frac{3}{7}$ to a fraction whose denominator shall be 35.
2. " $\frac{5}{8}$ " " " " " " 24.
3. " $\frac{9}{16}$ " " " " " " 48.
4. " $\frac{7}{15}$ " " " " " " 45.
5. " " " " " " " " 60.
6. " $\frac{19}{20}$ " " " " " " 100.
7. " $\frac{12}{5}$ " " " " " " 40.
8. " $\frac{5}{6}$ " " " " " " 18.
9. " $\frac{15}{17}$ " " " " " " 34.
10. " $\frac{11}{12}$ " " " " " " 84.

LESSON VIII.

53. To reduce two or more fractions to a common denominator. (That is, to the **Same** denominator).

Let us reduce $\frac{2}{3}$ and $\frac{4}{5}$ to a common denominator.



Remembering that by [(41)–(43)] fractions should be kept in their lowest, or as low, terms as possible, let AB and CD be two lines of the *same* length; and let AB be divided into 3 equal parts, and CD into 5 equal parts.

Then $AE=\frac{2}{3}$,
and $CF=\frac{4}{5}$.

Now the question is, what is the *smallest* number of equal parts into which A B and C D can be divided, so that the parts will be common to both lines. If each of the 3 equal parts of A B is divided into 5 equal parts,

$$A\,B=15 \text{ equal parts.}$$

Again, if each of the 5 equal parts of C D is divided into 3 equal parts,

$$C\,D=15 \text{ equal parts.}$$

But $A\,B=C\,D$.

$\therefore$ each equal part is $\frac{1}{15}$ of the *same*, or *equal* thing.

$\therefore$ $\frac{1}{15}$ is common to both A B and C D.

∴ by (52) reduce $\frac{2}{3}$ and $\frac{4}{5}$ to a fraction whose denominator is 15.

∵ $\frac{3}{3}=\frac{15}{15}$,

$\frac{1}{3}=\frac{1}{3}$ of $\frac{15}{15}=\frac{5}{15}$,

and $\frac{2}{3}=2\times\frac{5}{15}=\frac{10}{15}$.

Again ∵ $\frac{5}{5}=\frac{15}{15}$,

$\frac{1}{5}=\frac{1}{5}$ of $\frac{15}{15}=\frac{3}{15}$,

and $\frac{4}{5}=4=\frac{3}{15}=\frac{12}{15}$.

∴ **L. C. D.**† of $\frac{2}{3}$, $\frac{4}{5}=\frac{10}{15}$, $\frac{12}{15}$

This teaches that the least common denominator of two or more fractions is the least number that each one of the denominators will divide without a remainder.

When the denominators are *small*, their L. C. D. may be seen by **Inspection.** Let us notice this when the denominators are large.

54. Reduce $\frac{3}{8}$, $\frac{5}{24}$, and $\frac{7}{40}$ to their L. C. D.

Prime factors of $8=2\times2\times2$,

" " $24=2\times2\times2\times3$,

" " $40=2\times2\times2\times5$.

∴ the *least number* that 8, 24, and 40 will divide is

$$2\times2\times2\times3\times5=120.$$

∴ **L. C. D.** of 8, 24, 40=120.

∴ $\frac{3\times15^{*}}{8\times15}=\frac{45}{120}$,

$\frac{5\times5}{24\times5}=\frac{25}{120}$,

and $\frac{7\times3}{40\times3}=\frac{21}{120}$,

---

†L. C. D. denotes the least common denominator of two or more numbers.

*$\frac{3}{8}{\times}\frac{15}{15}$ should be read "$\frac{3}{8}$, each term multiplied by 15," &c. This can be analyzed by (52); but in (50) multiplying both terms of a fraction by the same thing does not change its value.

Note.—Separate the denominators into their prime factors, and take the product of all the prime factors *so many times as are equal to the highest number of times* any one of the prime factors has entered any one of the denominators. Above, 2 has entered 3 *times*, 3 *once*, and 5 *once*.

In the same manner, reduce the following to their L. C. D.:

1. $\frac{5}{16}$, $\frac{1}{12}$, $\frac{8}{15}$.
2. $\frac{17}{18}$, $\frac{22}{45}$, $\frac{19}{38}$, $\frac{5}{6}$.
3. $4\frac{5}{6}$,* $\frac{11}{12}$, $\frac{13}{14}$.
4. $\frac{25}{36}$, $\frac{3}{11}$, $\frac{12}{31}$, $\frac{2}{3}$.
5. $\frac{5}{6}$, $\frac{3}{4}$, $\frac{1}{2}$.
6. $\frac{7}{10}$, $2\frac{3}{4}$, $\frac{9}{14}$, $\frac{11}{18}$.
7. $\frac{3}{5}$, $\frac{7}{30}$, $\frac{2}{25}$.
8. $\frac{17}{72}$, $\frac{5}{24}$ $\frac{11}{42}$, $\frac{13}{80}$.
9. $\frac{6}{7}$, $\frac{3}{5}$.
10. $\frac{12}{13}$, $\frac{7}{8}$, $\frac{9}{15}$.

*Reduce to an improper fraction.

ADDITION OF FRACTIONS.

LESSON I.

55. When the denominators are **Alike.**

1. John paid $\frac{3}{8}$ of a cent for a stick of candy, and $\frac{7}{8}$ of a cent for a top; how much did he pay for both?

56. **Analysis:**

∵ the cost of the candy $=\frac{3}{8}$ of a cent,
and " " top $=\frac{7}{8}$ " "
" " both $=\frac{3}{8}+\frac{7}{8}$
$=\frac{10}{8}=1\frac{2}{8}$
$=1\frac{1}{4}$ cents.

Note.—When the denominators are **Alike**—that is, the **Same**—add the numerators. The sum of the numerators will be *so many* of the *equal* parts represented by the denominator. Above, the denominator is *eighths*. ∴ $\frac{10}{\text{eighths}}$.

Analyze and arrange the following:

2. A sick man walked $\frac{2}{3}$ of a mile on Monday, $\frac{4}{3}$ of a mile on Tuesday, $\frac{5}{3}$ of a mile on Wednesday, and $\frac{7}{3}$ of a mile on Thursday; how far did he walk in all?

3. A merchant has 5 remnants of ribbon: in the first is $\frac{5}{6}$ of a yard, in the second $\frac{1}{6}$ of a yard, in the third $\frac{7}{6}$ of a yard, in the fourrh $\frac{3}{6}$ of a yard, in the fifth $\frac{4}{6}$ of a yard; how many yards has he?

4. A farmer with his drill put in $\frac{3}{7}$ of an acre of wheat for one man, $\frac{5}{7}$ of an acre for another, and $\frac{2}{7}$ of an acre for another; how much did he put in for the three men?

5. Excavated $\frac{2}{12}$ of a cubic yard of earth for A, $\frac{5}{12}$ for B, $\frac{7}{12}$ for C; how much was excavated in all?

LESSON II.

56. When the denominators are **Unlike.**

1. In surveying a railroad, a corps advanced $\frac{3}{4}$ of a mile the first day, $\frac{5}{6}$ of a mile the second day, and $\frac{7}{8}$ of a mile the third day; how far had the corps advanced in the 3 days?

57. **Analysis:**

∵ the first day's advance $=\frac{3}{4}$ of a mile,
second " " $=\frac{5}{6}$ " " "
and third " " $=\frac{7}{8}$ " " "
three " " $=\frac{3}{4}+\frac{5}{6}+\frac{7}{8}$ of a mile.

But L. C. D. of the denominators, 4, 6, 8=24.

∴ $$\frac{3}{4}=\frac{3\times6}{4\times6}=\frac{18}{24},$$

$$\frac{5}{6}=\frac{5\times4}{6\times4}=\frac{20}{24},$$

and $$\frac{7}{8}=\frac{7\times3}{8\times3}=\frac{21}{24},$$

∴ the 3 days advance$=\frac{18}{24}+\frac{20}{24}+\frac{21}{24}$ of a mile
$=\frac{59}{24}=2\frac{11}{24}$ miles.

In the same manner, analyze and arrange the following:

2. Bought $\frac{2}{3}$ of a ton of guano from one company, $\frac{4}{5}$ of a ton from another company, and $\frac{3}{8}$ of a ton from an other; how much was bought from the three companies?

3. Collected $\frac{1}{2}$ of a dollar from one boy, $\frac{2}{3}$ of a dollar from another, $\frac{9}{10}$ of dollar from another, and $\frac{1}{5}$ of a dollar from another, for missions; how much did I collect?

4. Gave $\frac{5}{18}$ of my money for shoes, $\frac{1}{6}$ for suspenders, $\frac{1}{60}$ for a tie; how much of my money did I spend?

5. In making a fill worth $500, one cart moved $\frac{1}{5}$ of the dirt, one moved $\frac{1}{3}$, and one moved $\frac{1}{6}$ of the dirt; how much did the 3 carts move? What was it worth? and what did each cart-man get?

6. A man invests $\frac{1}{3}$ of his money in stock, $\frac{1}{4}$ in a railroad, $\frac{1}{6}$ in land; how much did he invest in all?

LESSON III.

58. To add **Mixed** numbers.

1. Threshed $30\frac{1}{3}$ bushels of wheat, for A, $55\frac{1}{2}$ bushels for B, and $45\frac{3}{4}$ bushels for C; how many bushels were threshed in all?

59. **Analysis:**

$\because$ No. bu. threshed for A$=30\frac{1}{3}$,
" " B$=55\frac{1}{2}$,
" " C$=45\frac{3}{4}$,
and " " all$=30\frac{1}{3}+55\frac{1}{2}+45\frac{3}{4}$ bushels
$=\frac{91}{3}+\frac{111}{2}+\frac{183}{4}$, [reducing to an imp.frac.]
$=\frac{364}{12}+\frac{666}{12}+\frac{459}{12}$, [reducing to L. C. D.]
$=\frac{1579}{12}$
$=131\frac{7}{12}$.

60. It is recommended that the whole numbers and fractions be added **Separately**; especially when the whole numbers are **Large**. Taking the same problem,

Analysis:

$\because$ No. bu. threshed for A$=30\frac{1}{3}$,
" " B$=55\frac{1}{2}$,
" " C$=45\frac{3}{4}$,
" " all$=130+\frac{1}{3}+\frac{1}{2}+\frac{3}{4}$
$=130+\frac{4}{12}+\frac{6}{12}+\frac{9}{12}$, [reducing to L.C.D
$=130+\frac{19}{12}$, [adding the fractions],
$=130+1\frac{7}{12}$, [reducing $\frac{19}{12}$ to mixed number],
$=131\frac{7}{12}$, same as above.

Note.—In the work of the following problems, let either of the above *arrangements* be strictly observed; for by so doing logical habits of mind flow smoothly.

2. Tomlin of Archdale shipped from Trinity one box of shoes weighing $224\frac{2}{3}$ ℔., one box weighing $500\frac{3}{4}$ ℔, one box weighing $356\frac{4}{5}$ ℔, one box weighing $425\frac{1}{2}$ ℔, one box weighing $144\frac{7}{8}$ ℔; how many ℔s. of freight did he ship?

3. Bought 4 carpets. The first contained $25\frac{7}{8}$ yards, the second contained $18\frac{4}{5}$ yards, the third contained $22\frac{1}{2}$ yards, the fourth contained $22\frac{1}{6}$ yards; how many yards did I buy?

4. In one pile of lumber are $400\frac{5}{12}$ feet, in another pile are $600\frac{9}{10}$ feet, in another are $756\frac{3}{4}$ feet; how many feet of lumber in the three piles?

5. Find the sum of $\frac{7}{8}+\frac{2}{5}+3\frac{3}{4}+\frac{9}{10}+8\frac{1}{2}$.

SUBTRACTION OF FRACTIONS.

LESSON I.

---

61. When the denominators are **Alike.**

1. Bought $\frac{7}{8}$ of a bushel of clover seed, and sold $\frac{5}{8}$ of a bushel; how much had I left?

62. **Analysis:**

∴ what I had $= \frac{7}{8}$ of a bu.,
and " sold $= \frac{5}{8}$ "
what I had left $= \frac{7}{7} - \frac{5}{8} = \frac{2}{8}$
$= \frac{1}{4}$ of a bu. [reducing to lowest terms].

2. In an excavation $\frac{7}{18}$ of a ton of rock is to be removed. $\frac{4}{18}$ of a ton having been removed; what part of a ton is yet to remove?

3. If I have $\frac{13}{15}$ of a cord of wood, and sell $\frac{8}{15}$ of a cord; how much have I left?

4. $\frac{18}{24}$ of a mile of road is to be built. $\frac{11}{24}$ of a mile has been built; what part of a mile remains to be built?

5. Have $\frac{14}{17}$ of a ton of Guano. Sold $\frac{5}{17}$ of a ton; what part of a ton remains?

LESSON II.

---

63. When the denominators are **Unlike**.

1. Purchased $\frac{11}{15}$ of a bale of feed, and sold $\frac{2}{3}$ of a bale; what part of a bale had I left?

69. **Analysis:**

what I had $=\frac{11}{15}$ of a bale,
and " sold $=\frac{2}{3}$ " "
" had left$=\frac{11}{15}-\frac{2}{3}$
$=\frac{11}{15}-\frac{10}{15}$, [reducing to L. C. D.]
$=\frac{1}{15}$ of a bale.

2. Had $\frac{17}{24}$ of a line of road to grade. Let out $\frac{7}{18}$ to a sub-contractor; what part of the whole line did I yet have?

3. If the grade of a road for a certain distance was to be $\frac{28}{30}$ of a foot; but the engineer found that this was $\frac{2}{45}$ of a foot too much. What part of a foot was the grade for this distance?

4. A township was taxed $\frac{9}{10}$ of a cent on the dollar for some improvements; but afterward was taxed $\frac{7}{8}$ of a cent. By how much too great was the former tax?

5. Bought $\frac{9}{42}$ of a yard of carpet. Sold $\frac{1}{60}$ of a yard; how much had I left?

LESSON III.

65. To take a **Fraction** from a **Whole** number.

1. Had a piece of goods containing 36 yards, from which I sold $\frac{7}{8}$ of a yard; how much had I left?

66. **Analysis :**

∵ what I had $=36$ yards,
and " sold $=\frac{7}{8}$ of a yard,
" had left $=36-\frac{7}{8}$
$=35\frac{8}{8}-\frac{7}{8}$
$=35\frac{1}{8}$ yards.

Note.—36 yds. $=35\frac{8}{8}$ yds., a *form* from which $\frac{7}{8}$ can be taken. ∴ always change the whole number to this form.

2. Received $22\frac{1}{3}$ bales of feed from one firm, and $25\frac{2}{3}$ bales from an other; sold $\frac{4}{5}$ of a bale. How much had I left?

3. From 52 ℔s. of salt, I sold $\frac{1}{2}$ of a ℔.; how much had I left.

4. From 6 yards of muslin, I sold $\frac{5}{6}$ of a yard; how much had I left?

5. A line 80 chains in length was shortened by $\frac{9}{10}$ of chain; what then was the length of the line?

LESSON IV.

67. To take a **Mixed** number from a **Mixed** number.

1. From a pile of lumber containing $327\frac{7}{8}$ feet, I sold $125\frac{2}{3}$ feet; how much had I left?

62. **Analysis.**

∵ what I had $=325\frac{7}{8}$ feet,
and " sold $=125\frac{2}{3}$ "
" had left $=325\frac{7}{8}-115\frac{2}{3}$
$=325\frac{21}{24}-125\frac{16}{24}$ [red. frac to L. C. D.]
$=200\frac{5}{24}$ feet.

Note.—Mixed numbers could be reduced to improper fractions; but this is not recommended. If the numbers are large, it is better to subtract as above.

Let us take another example in which the fraction connected with the subtrahend* is larger than the fraction connected with the minuend, number made less.

2. Had $250\frac{1}{3}$ feet of lumber, and sold $75\frac{7}{8}$ feet; how much lumber had I left?

---

*The number to be subtracted.

69. **Analysis:**

$$\begin{aligned}
\therefore\quad &\text{what I had} &&= 250\tfrac{1}{3}\text{ feet,}\\
\text{and}\quad &\text{`` `` sold} &&= 75\tfrac{7}{8}\text{ ``}\\
&\text{`` `` had left} &&= 250\tfrac{1}{3}-75\tfrac{7}{8}\\
& &&= 250\tfrac{8}{24}-75\tfrac{21}{24},\\
& &&= 249+\tfrac{24}{24}+\tfrac{8}{24}-75\tfrac{21}{24},\\
& &&= 249\tfrac{32}{24}-75\tfrac{21}{24},\\
& &&= 174\tfrac{11}{24}.
\end{aligned}$$

Note.—Since $\frac{21}{24}$ is greater than $\frac{8}{24}$, 1$(=\frac{24}{24})$ must be taken from 250, leaving 249, and added to $\frac{8}{24}$ making $\frac{32}{24}$. Now $\frac{21}{24}$, which belongs to the subtrahend, can be taken from $\frac{32}{24}$.

3. John bought a sled for $\frac{9}{10}$ of a dollar, not pleased with his purchase, he sold it, and lost $\frac{1}{3}$ of a dollar. How much did he get for the sled?

4. If you give $7\frac{1}{2}$ oranges to one boy and $2\frac{1}{3}$ oranges to another, how much more do you give to one boy than to the other?

5. A man bought a book, the price of which was $4\frac{5}{8}$ dollars and paid $1\frac{7}{8}$ dollars; how much did he still owe?

6. If a laborer can dig $18\frac{3}{4}$ cubic yards of earth in one day; but $11\frac{2}{3}$ yards are already removed; how much is still to be moved?

7. A grocer purchased a box of eggs, for which he paid $3\frac{1}{5}$ dollars, and sold them for $4\frac{3}{4}$ dollars; how much did he make?

MULTIPLICATION OF FRACTIONS.

LESSON I.

70. To multiply a **Fraction** by a **Whole** number.

1. If 1 pencil cost $\frac{4}{15}$ of a cent; what will 5 pencils cost?

71. **Analysis:**

∵ the cost of 1 pencil $=\frac{4}{15}$ of a cent,
" " 5 pencils $=5\times\frac{4}{15}=\frac{20}{15}$
$=1\frac{5}{15}$
$=1\frac{1}{3}$ cents. [reduce to L. T.]

Note.—Instead of $5\times\frac{4}{15}=\frac{20}{15}$ we could have made $5\times\frac{4}{15}=\frac{5\times4}{15}=\frac{4}{3}=1\frac{1}{3}$; *which should always be done when the multiplier will exactly divide the denominator.*

72. **Analysis:**

∵ the cost of 1 pencil $=\frac{4}{15}$ of a cent,
" " 5 pencils $=5\times\frac{4}{15}$ of a cent,
$\frac{5\times4}{15}=\frac{4}{3}$
$=1\frac{1}{3}$ cents.

∴ by [(71)-(72)] it is clearly demonstrated that to *mul-*

tiply a fraction, either multiply the numerator or divide the denominator.

2. Bought 8 pounds of butter at $\frac{1}{5}$ of a dollar a pound, 22 pounds of ham at $\frac{1}{8}$ of a dollar a pound, and 20 bushels of meal at $\frac{2}{3}$ of a dollar a bushel; what did it all cost?

73. **Analysis:**

∵ the cost of 1 pound of butter $=\frac{1}{5}$ of a dollar,
" 8 pounds " $=8\times\frac{1}{5}=\frac{8}{5}$
$=1\frac{3}{5}$ dollars.

∵ the cost of 1 lb. ham $=\frac{1}{8}$ of a dollar,
" 22 lbs. " $=22\times\frac{1}{8}=\frac{22}{8}$
$=2\frac{6}{8}$
$=2\frac{3}{4}$ dollars.

∵ the cost of 1 bu. meal $=\frac{2}{3}$ of a dollar,
" 20 bus. " $=20\times\frac{2}{3}$
$=\frac{40}{3}=13\frac{1}{3}$ dollars.

∴ ∵ the cost of the butter $=1\frac{3}{5}$ dollars,
" " " ham $=2\frac{3}{4}$ "
and " " " meal $=13\frac{1}{3}$ "
" " " all $=16+\frac{3}{5}+\frac{3}{4}+\frac{1}{3}$
$=16+\frac{36}{60}+\frac{45}{60}+\frac{20}{60}$
$=16+\frac{101}{60}$
$=16+1\frac{41}{60}$
$=17\frac{41}{60}$ dollars.

Note.—Attend to all the details of analysis. The *valuable fruits* of mental discipline will be the reward. As you grow strong in mathematical thinking you can omit some *details*, and abridge work.

3. Shipped 24 doz. eggs at $\frac{1}{8}$ of a dollar per doz., 25 chickens at $\frac{3}{5}$ of a dollar a piece; and received a 20-dollar bill; what change should I return?

4. Bought 24 yards of carpet at $\frac{2}{3}$ of a dollar a yard; what did the carpet cost?

5. Carried 50 bu. corn to mill. The miller ground for the tenth. How many bushels of meal did I bring back? and what was it worth at $\frac{2}{5}$ of a dollar a bu.?

6. If I had sold the meal to A at the obove price; how much would A have gained if he had sold the meal to B at $\frac{2}{3}$ of a dollar per bu.?

7. What would it cost to remove 851 cubic yards of earth at $\frac{9}{50}$ of a dollar per c. yd.?

8. Bought 38 mackerel at $\frac{1}{4}$ of a dollar a piece, and sold them at $\frac{3}{10}$ of a dollar a piece; what did I gain?

9. Shipped 8 boxes of shoes, 200 pair in each box, at $\frac{3}{4}$ of a dollar a pair; what were the shoes worth?

10. What are 45 gal. molasses worth at $\frac{1}{3}$ of a dollar per gal.?

11. Paid for 200 bundles of fodder at $\frac{2}{3}$ of a cent a bundle; what did it cost me?

12. If 5 men in 4 hours can move $\frac{2}{3}$ of a ton of rock, how much rock can 7 men in 2 hours move?

LESSON. II.

74. To multiply a **Mixed** number by a Whole number.

1. At $3\frac{4}{5}$ dollars per bushel, what will 12 bushels of dried peaches cost?

75. **Analysis:**

∵ the cost of 1 bu. peaches $=3\frac{4}{5}=\frac{19}{5}$ dollars,
" " 12 " $=12\times\frac{19}{5}$ dollars,
$=\frac{228}{5}$
$=45\frac{3}{5}$ dollars.

Note.—By reducing the *mixed* number to an *improper* fraction, (70) applies. Some prefer to multiply the whole numbers and fractions separately. It, however, is a question of convenience. Let us take the same problem.

76. **Analysis:**

∵ the cost of 1 bu. peaches $=3\frac{4}{5}$ dollars,
" " 12 " $=12\times3\frac{4}{5}$
$=36\frac{48}{5}$
$=36+9\frac{3}{5}$
$=45\frac{3}{5}$ dollars.

2. Bought 50 bushels of wheat at $1\frac{1}{2}$ dollars per bu.; what is the cost of the 50 bushels?

3. Had shipped from Richmond 1,000 lbs. of ham at $8\frac{3}{4}$ cents a pound, and sold it at $12\frac{1}{2}$ cents a pound; what did I make?

4. If a wheat drill in 1 day can set $18\frac{3}{4}$ acres in wheat, how many acres can it set in 6 days?

5. How much must I pay for mowing 12 acres of hay at $1\frac{2}{5}$ dollars per acre?

6. Sold 20 bushels of potatoes at $1\frac{5}{6}$ dollars per bu.; what did I receive for them?

7. Bought 725 yards of cloth at $2\frac{4}{5}$ dollars per yard; what did I pay for it?

LESSON III.

77. To multiply a **Whole** number by a fraction.

1. What must I pay for $\frac{3}{4}$ of a ton of guano, if a ton cost 20 dollars?

78. **Analysis:**

∵ the cost of 1 ton*$=\frac{4}{4}=$20 dollars,
" " $\frac{1}{4}$ " $=\frac{1}{4}$ of $20=5$ dollars,
and " " $\frac{3}{4}$ " $=3\times5=15$ dollars.

2. If 1 lb. of butter is worth 18 cents; what is $\frac{2}{3}$ of a lb. worth?

3. If a can of lard is worth 15 dollars; what is $\frac{4}{5}$ of a can worth?

4. Bought a barrel of molasses for 45 dollars, upon

---

*Change 1 to *fourths*, because the required is in the denomination of fourths.

examination $\frac{2}{7}$ had leaked away; what was the remainder worth ?

5. Invested $\frac{1}{2}$ of my money in stocks, $\frac{1}{4}$ in real estate, $\frac{1}{6}$ in railroad stock, and had 3,000 dollars left. How much money had I ?

6. In making an excavation from each end of a railroad cut, one party had gone $\frac{2}{7}$ of the whole way, and the other party had gone $\frac{1}{3}$ of the way. By measurement 1,600 feet was between them. What was the whole length of the cut ?

7. If the whole cut was worth 450 dollars; what was the excavation at this time worth ?

8. If all my money amounts to 800 dollars; what is $\frac{3}{8}$ of it ?

9. A sack of flour is worth 3 dollars. What is $\frac{3}{4}$ of it worth ?

10. A man ships a cargo of sugar, worth 1500 dollars, to New York. When it reaches New York, $\frac{4}{5}$ of it is found good. How much should the man receive from New York?

LESSON IV.

79. To multiply a **Fraction** by a **Fraction.**

1. Owned $\frac{6}{7}$ of a cotton mill, and sold $\frac{2}{3}$ of my share to Smith. What part of the mill did Smith own?

80. **Analysis.**

$\because$ $\frac{3}{3}$, or all, of my share $=\frac{6}{7}$ of the whole mill,
$\frac{1}{3}$ " " $=\frac{1}{3}$ of $\frac{6}{7}=\frac{2}{7}$ of the whole mill,
and $\frac{2}{3}$ " " $=2\times\frac{2}{7}=\frac{4}{7}$ " " "

Note.—In taking $\frac{1}{3}$ of $\frac{6}{7}$, we divide 6, the numerator, by 3, which gave us $\frac{2}{7}$. We could have done thus:

81. **Analysis:**

$\because$ $\frac{3}{3}$, or all, of my share $=\frac{6}{7}$ of the mill,
$\frac{1}{3}$ " " " $=\frac{1}{3}$ of $\frac{6}{7}=\frac{6}{21}$ of the mill,
and $\frac{2}{3}$ " " " $=2\times\frac{6}{21}=\frac{12}{21}$ " " "
$=\frac{4}{7}$, [reducing $\frac{12}{21}$ to L. T.]

$\therefore$ by [(80)-(81)] it is plain that a fraction can be divided by *dividing* the *numerator*, or *multipling* the *denominator*.

Note.—In all cases in which the numerator *can de divided by the divisor without a remainder* it *should be done.* When the numerator *cannot so be divided,* multiply the denominator.

To make this plain, let us take an example in which the numerator cannot be divided.

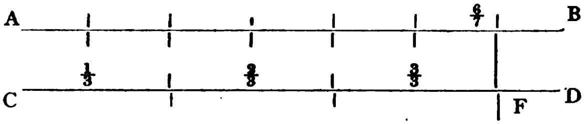
2. Bought $\frac{5}{8}$ of a yard of Ribbon, and sold $\frac{6}{7}$ of my share. What part of the whole ribbon did I sell?

82. **Analysis:**

$\because$ $\frac{7}{7}$, or all, of my share $=\frac{5}{8}$ of the whole ribbon,
$\frac{1}{7}$ " " $=\frac{1}{7}$ of $\frac{5}{8}=\frac{5}{55}$ of the W. R.
and $\frac{6}{7}$ " " $=6\times\frac{5}{56}=\frac{6\times5}{56}$
$=\frac{3\times5}{28}=\frac{15}{28}$ of the ribon.

Before proceeding further I wish to explain a fact connected with this subject, which fact is not always clear to some minds. The fact referred to is, how can $\frac{3}{3}=\frac{6}{7}$ (79)? or how can a whole number=a fraction? Let us take one under (79.)

83. Graphical Analysis.



Let A B represent the *whole* value of the mill, divided into 7 equal parts.

Then my share A E=$\frac{6}{7}$ of the whole mill. (7.)

Also let C D be equal in lenth to A B. Placing their ends even,

A E=C F

But my share C F=$\frac{3}{3}$, or *all of itself.* (2.)

.·. by (1) and (2), $\frac{3}{3}$*=$\frac{6}{7}$

Note.—Both $\frac{3}{8}$ and $\frac{6}{7}$ are *my share;* one is the *whole* of *itself*, the other is $\frac{6}{7}$ of the *whole* mill.

3. Asked $\frac{3}{5}$ of a dollar for corn per bushel; but was compelled to take $\frac{2}{3}$ of my asking price. What part of a dollar did I get per bushel? What did I lose on a bushel? What did I lose on 300 bushels?

---

*This is not true with reference to the same unit.

4. Sold $\frac{2}{3}$ of my share in a farm worth 400 dollars. I owned $\frac{4}{5}$ of the farm. What did I get?

5. Bought $\frac{9}{10}$ of an interest in a company, and sold $\frac{1}{3}$ of my share for 600 dollars. What was the capital stock?

6. What part of the capital had I left, and what was it worth?

7. Contracted to build $\frac{7}{8}$ of a line of road worth 15,000 dollars. I sub-let $\frac{1}{2}$ of my share to A, who sub-let $\frac{2}{3}$ of his part to B. What part of the road did each build? What part of the money did each get?

84. **Analysis:**

$\because$ $\frac{2}{2}$, or all, of my share $=\frac{7}{8}$ of the whole line,
$\frac{1}{2}$ “ “ $=\frac{1}{2}$ of $\frac{7}{8}=\frac{7}{16}$ of whole line, A's.
$\because$ $\frac{3}{3}$, or all of A's share, $=\frac{7}{16}$ of the whole line,
$\frac{1}{3}$ “ “ “ $=\frac{1}{3}$ of $\frac{7}{16}=\frac{7}{48}$ of whole line,
and $\frac{2}{3}$ “ “ “ $=2\times\frac{7}{48}=\frac{7}{24}$ “ “ “ , B's.
Then $\because$ at first my share $=\frac{7}{8}$ of the line,
and by selling $\frac{1}{2}$ of it, $=\frac{7}{16}$ “ “ , A's share,
finally my share $=\frac{7}{8}-\frac{7}{16}$
$=\frac{14}{16}-\frac{7}{16}=\frac{7}{16}$ of the line.
But $\because$ at first A's share $=\frac{7}{16}$ of the whole line,
and by selling $\frac{2}{3}$ of it $=\frac{7}{24}$ “ “ “ , B's share,
finally A's share $=\frac{7}{16}-\frac{7}{24}$
$=\frac{21}{48}-\frac{14}{48}=\frac{7}{48}$ of the line.
$\therefore$ my share of money $=\frac{7}{16}$ of 15,000 $=\frac{105000}{16}=6562\frac{1}{2}$,
A's “ “ $=\frac{7}{48}$ “ $=\frac{105000}{48}=2187\frac{1}{2}$,
and B's “ “ $=\frac{7}{24}$ “ $=\frac{105000}{24}=4375$.

Note.—By some authors the above has been defined as compound fractions.

8. Rented my farm to A, who was to do all the labor, and give me $\frac{1}{3}$ of all raised, Having husked the corn, he told me that he had already used 24 bushels of corn. How much should I take off of the pile before beginning the division?

9. Rented a tobacco farm from Smith for $\frac{3}{4}$ of all tobacco raised. I afterwards took Jones into co-partnership at $\frac{4}{5}$ of my part. The whole yield was 1800 lbs., worth 40 cents a lb. How much money did each get?

10. If a fish cost $2\frac{1}{5}$ cents, and a nutmeg $\frac{4}{5}$ of the fish, what is the cost of the nutmeg?

DIVISION OF FRACTIONS.

LESSON I.

85. To divide a **Fraction** by a **Whole** number.

1. If 5 boys get $\frac{15}{16}$ of a dollar; what does each boy get?

86. **Analysis:**

$\because$ what 5 boys get $=\frac{15}{16}$ of a dollar,
" 1 boy gets $=\frac{1}{5}$ of $\frac{15}{16}$ of a dollar.
$=\frac{3}{16}$ of a dollar.

Note.—Instead of *dividing* the numerator in the above example by 5, we could have *multiplied* the denominator. Thus:

87. **Analysis:**

$\because$ what 5 boys get $=\frac{15}{16}$ of a dollar,
" 1 boy gets $=\frac{1}{5}$ of $\frac{15}{16}$ of a dollar,
$=\frac{15}{80}=\frac{3}{16}$ [reducing] same as above.

$\therefore$ by [(86)-(87)] it is clear that to divide a fraction, either *divide* the numerator, or *multiply* the denominator. Always *divide* the numerator if it can be done.

2. Owned $\frac{7}{8}$ of a farm, and divided it equally among three men; what part of the farm did each get?

3. Had shipped me $2\frac{1}{3}$ tons of phosphate. Divided it equally among myself and 3 other men; what was each one's part?

4. If 4 men own $\frac{2}{3}$ of $\frac{7}{8}$ of a mine, and sell $\frac{4}{8}$ of what they own, how much has each man left?

5. If the whole mine was valued at $250,000, what is the value of each one's share?

6. 9 men own $\frac{8}{10}$ of the stock in a school building. By fire the building is damaged $\frac{1}{3}$ of its value. Then 2 of the men draw out. Taking with them $\frac{1}{4}$ of first remainder. What is each one's part in the stock?

7. If 3 yards of ribbon are worth $\frac{12}{13}$ of a dollar, what is 1 yard worth?

LESSON II.

88. To divide a **Whole** number by a **Fraction.**

1. At $\frac{2}{3}$ of a dollar a yard, how much cloth can be bought for 4 dollars?

89. **Analysis:**

∵ $\frac{2}{3}$ of a dollar $=1$ yard of cloth,
$\frac{1}{3}$ " " $=\frac{1}{2}$ of 1 yard $=\frac{1}{2}$ of a yard,
$\frac{3}{3}$, or 1 dollar $=3\times\frac{1}{2}$ of a yard $=\frac{3}{2}$ yards,
and 4 dollars $=4\times\frac{3}{2}$ yards $=6$ yards.

2. If $\frac{3}{4}$ of my money is 600 dollars; how much money have I?

3. At $\frac{1}{2}$ of $\frac{2}{3}$ of a dollar a yard, how much cassimere can be bought for 18 dollars?

4. At $\frac{1}{8}$ of a dollar a pound, how much butter can be bought for 3 dollars?

5. My agent collected 600 dollars which was $\frac{12}{13}$ of the money due me. What was due me?

6. In one field is $\frac{1}{3}$ of my sheep, in another is $\frac{1}{4}$ of my sheep, in another is $\frac{1}{5}$ of my sheep, and in the fourth field are 91 sheep. How many sheep had I?

7. I ask $\frac{3}{5}$ of a dollar for a bushel of corn, but was compelled to take $\frac{4}{5}$ of my asking price less; how many bushels must I sell in order to get 30 dollars?

8. Rented my cotton farm so that I got $\frac{1}{4}$. In the first ginning my renter took all, which was 1500 lbs. How much must I take out of the next ginning before he takes any?

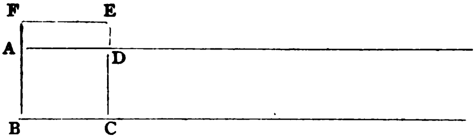
9. How many sacks, each containing $1\frac{7}{8}$ yards, can be made out of 30 yards?

10. A, B and C bought a farm for which they paid 4500 dollars. A bought $\frac{1}{3}$, B bought $\frac{3}{5}$ and C the remainder. What did each pay?

11. At $1\frac{2}{3}$ dollars an acre, how much land was in the above farm, and how much did each own?

12. If carpeting is $\frac{3}{4}$ of a yard wide, how many yards will cover 30 square yards?

90. **Analysis:**



Let A B denote the width of this carpet, which is ¾ of a yard. Then if B C is 1 yard in length, the figure A, B, C, D has ¾ of a square yard in it. It lacks the ¼ of a yard which is in A D E F. It is plain, therefore, that to get ¾ of a *square* yard, we must go 1 yard in *length*, or from B to C.

∴ ∵ to get ¾ of a square yd, =1 yd. in length,
" " ¼ " " " =⅓ of 1 yd. in length
=⅓ of a " "
" " 4/4, or a whole sq. yd.=4×⅓=4/3 of a yd. in l'g'th.
and " " 30 square yds.=30×4/3=40 yds. in length.

13. How much brown sheeting ⅞ of a yard wide will cover 18 square yards?

14. Sold 40 lbs. of lard at 12½ cents a pound, and took my pay in calico at 6¼ cents a yard. How many yards could I buy?

15. How many feet of plank in length, if the plank are ⅔ of a foot wide, will be required to floor 50 square feet?

LESSON III.

91. To divide a **Fraction** by a **Fraction.**

1. At $\frac{3}{5}$ of a dollar a bushel, how many bushels of corn can I buy for $4\frac{4}{5}$ dollars?

92. **Analysis:**

∵ $\frac{3}{5}$ of a dol. $=$ 1 bushel of corn,
$\frac{1}{5}$ " " " $=\frac{1}{3}$ of 1 bu. corn $=\frac{1}{3}$ of a bu.,
$\frac{5}{5}$, or 1 dol. $=5\times\frac{1}{3}=\frac{5}{3}$ bushels,
and $\frac{24}{5}$ of a " $=24\times\frac{5}{3}$ bushels
$=40$ bushels.

Note.—Here $4\frac{4}{5}$, the amount with which we buy, is in kind like the *known* term $\frac{3}{5}$: namely, both are in *fifths.* Let us take another example in which this is not the case.

2. At $\frac{3}{8}$ of a dollar a bushel, how many bushel of potatoes can be bought for $5\frac{2}{3}$ dollars?

93. **Analysis:**

∵ $\frac{3}{8}$ of a dol. $=$ 1 bu. potatoes,
$\frac{1}{8}$ " " " $=\frac{1}{3}$ of 1 bu. potatoes
$=\frac{1}{3}$ bu.,
$\frac{8}{8}$, or 1 dol. $=8\times\frac{1}{3}$ bu. $=\frac{8}{3}$ bu.,
$\frac{1}{3}$ of a " $=\frac{1}{3}$ of $\frac{8}{3}$ bu. $=\frac{8}{9}$ bu.,
and $\frac{17}{3}$ " " " $=17\times\frac{8}{9}=\frac{136}{9}$
$=15\frac{1}{9}$ bushels.

3. How many days at $\frac{3}{4}$ of an acre a day will it take to plough $6\frac{4}{8}$ acres?

4. If $50\frac{1}{2}$ cubic yards can be removed per day, how many days will it require to move $756\frac{3}{4}$ cubic yds.?

5. Sold $23\frac{1}{2}$ lbs. of ham at $12\frac{1}{2}$ cents a pound, and took my pay in sugar at $7\frac{2}{3}$ cents a pound; how many pounds of sugar did I buy?

6. If A can do a piece of work in 6 days, B in 8 days, C in 10 days, what should each man receive if they work together and the work is worth 24 dollars?

94. **Analysis:**

Let 1 = the work done.

$\because$ part done by A in 6 days $=1$, or the work,

“ “ “ 1 day $=\frac{1}{6}$ of the work, (1).

$\because$ part done by B in 8 days $=1$, or the work,

“ “ “ 1 day $=\frac{1}{8}$ of the work, (2).

$\because$ part done by C in 10 days $=1$, or the work,

“ “ “ 1 day $=\frac{1}{10}$ of the work, (3).

$\therefore$ the part of the work done

by A, B, C, in 1 day $=\frac{1}{6}+\frac{1}{8}+\frac{1}{10}$

$=\frac{47}{120}$ of the work.

$\therefore\ \because$ to do $\frac{47}{120}$ of the work by A, B, C $=1$ day,

“ $\frac{1}{120}$ “ “ “ “ $=\frac{1}{47}$ of $1=\frac{1}{47}$ of a day

and “ $\frac{120}{120}$, the whole work $=120\times\frac{1}{47}$

$=\frac{120}{47}$ days.

Now $\because$ part done by A in 1 day $=\frac{1}{6}$ of the work, [from (1)],

“ “ “ $\frac{1}{47}$ of a day $=\frac{1}{47}$ of $\frac{1}{6}=\frac{1}{282}$ of the work,

and “ “ “ $\frac{120}{47}$ “ days $=120\times\frac{1}{282}$ of the work

$=\frac{120}{282}=\frac{60}{141}$ of the work.

$\therefore$ A's part of the money $=\frac{60}{141}$ of $24=\frac{1440}{141}$

$=10\frac{30}{141}$ dollars.

$\because$ part done by B in 1 day $=\frac{1}{8}$ of the work, [from (2)],

“ “ “ $\frac{1}{47}$ of day $=\frac{1}{47}$ of $\frac{1}{8}=\frac{1}{376}$ of the work,

and “ “ “ $\frac{120}{47}$ days $=120\times\frac{1}{376}$ of the work

$=\frac{120}{376}=\frac{30}{94}=\frac{15}{47}$ of the work.

$\therefore$ B's part of the money $=\frac{15}{47}$ of $24=\frac{360}{47}$

$=7\frac{31}{47}$ dollars.

In the same way find C's part.

7. A can do a piece of work in 8 days, B in 10 days, C in 15 days. They all work together for 2 days, when A and B leave C to finish the work. What should each receive if the work is worth 20 dollars?

8. If an engineer corps can make $1\frac{2}{3}$ miles a day, how many days will it take them to make $11\frac{4}{5}$ miles?

9. Asked for flour per barrel $\frac{5}{6}$ of what it cost me; was forced to take this made less by $\frac{2}{3}$ of it, and received 2 dollars per barrel. What did the flour cost me per barrel?

10. If A can do $\frac{5}{9}$ of a piece of work in 10 days, and B can do the whole work in 15 days, and the work is worth 25 dollars, what part of the money should each get if they work together?

11. How many yds. of carpeting will be required to cover 44 square yards if the material is $\frac{7}{8}$ of a yard wide?

LESSON IV.

REVIEW.

1. Sold $12\frac{1}{2}$ bushels of wheat to 1 man, $25\frac{1}{4}$ bushels to another, $\frac{3}{4}$ of a bushel to another; how much did I sell? What was it worth at $1\frac{2}{3}$ dollars per bushel?

2. The man to whom I sold the wheat lost $6\frac{5}{6}$ bushels, and sold the remainder at $\frac{2}{3}$ of its cost. What was his total loss?

3. Sold a barrel of flour for $\frac{1}{3}$ of $\frac{4}{5}$ of its original cost, and received $2\frac{1}{2}$ dollars. What was its original cost?

4. Of a carpet containing 24 yards $\frac{7}{8}$ of it got burnt. How many yards should I order, if the material is $\frac{3}{4}$ of a yard wide?

5. A, who had $\frac{7}{8}$ of a barrel of sugar, sold me $\frac{3}{4}$ of a barrel. The remainder he sold a poor widow for 2 dollars which was cost. What was the cost of the whole barrel?

6. Contracted to remove 6000 cubic feet of earth at 18 cents per 27 solid feet. With 10 men in 4 days 2000 c. feet were removed. Then 4 men withdrew; in how many days will the remainder of the men remove the remainder of the earth? How much did each man get?

7. Bought 40 gallons of molasses at $\frac{7}{8}$ of a dollar per gallon. Having lost 10 gallons, at what price per gal. must I sell the remainder to make 5 dollars on the outlay?

8. A, B and C start at the same point to walk around an island 72 miles in circumference. A walks 10 miles per day, B walks 12, and C 14. In how many days will they all be together, and where will it be?

Note.—Let this be solved by analysis.

9. Exchanged 40 bu. of corn at $\frac{2}{3}$ of a dol. per bushel, for sugar at $\frac{1}{8}$ of a dollar a lb. How many pounds did I get?

10. A ship will fill its hold of 25,000 gallons in 8 hours. If the leak gains 250$\frac{3}{4}$ gallons on the pump per hour, how many hours can the crew sustain itself?

11. A rents his land to B to sow in wheat. B was to furnish labor and seed wheat, and take $\frac{2}{3}$ of the grain.

but when seeding time came, B did not have the seed wheat. A, therefore, furnished the seed wheat, which entitled him to half. How should the wheat be divided?

12. If the crop makes 72 bushels, how much does each get?

13. Bought 500 feet of lumber at $\frac{3}{4}$ of a dollar per hundred, paid $\frac{1}{2}$ dollar for draying, lost $\frac{1}{2}$ of $\frac{2}{3}$ of the lumber by fire; at what price per hundred must I sell the remainder to lose nothing?

14. If $\frac{3}{4}$ of a pound of butter cost 15 cents, what would 12 lbs. cost?

15. Cut 8 coats from a piece of cloth, after which it measured 15 yards. Then I cut 5 pair of pants, which took $\frac{1}{2}$ as much as the coats, when I found $\frac{1}{2}$ the piece of cloth remaining. How many yards were in it at first?

95. It was stated [20] that the manner in which the **Unit** was divided determines the kind of fraction.

96. The **Unit** may be divided by the **Scale** of **Ten**; hence, **Decimal Fractions.**

97. A **Decimal Fraction**, therefore, is a fraction derived from the **Unit** by dividing the **Unit** by the **Scale of Ten.**

98.
$\frac{1}{10}$ is written .1,
$\frac{2}{10}$ “ “ .2,
$\frac{3}{10}$ “ “ .3,
$\frac{4}{10}$ “ “ .4,
$\frac{1}{100}$ “ “ .01, &c.

The point placed before decimals is called the *decimal* point.

99. The *first* place on the right of the decimal point is *tenths;* the *second* place is *hundredths;* the *third* place is *thousandths;* &c.

100. .1 is read *one tenth,*
.75 " " *seventy-five hundredths,*
.256 " " *two hundred and fifty-six thousandths,*
.75347 is read 75 347 hundred-thousandths.

101. **Whole** numbers and **Decimals** may be written together with the whole number on the left of the decimal point and the decimal places on the right of the decimal point. Thus:

25. 375.

102. In adding Decimals, Units of the *same order* should be placed under each other.

Note.—Having assumed that this has been taught under the subject of addition, it will not be treated here. If, however, addition of whole numbers and decimals is not sufficiently understood, the teacher can furnish black-board exercises till the subject is understood. The same applies to subtraction of decimals.

103. By (98) it is seen that every decimal fraction has a denominator understood. This denominator is 1 with as many *zeros* annexed as are equal to the places of decimals.

104. To multiply a Decimal by a Decimal.

$$
\begin{aligned}
&.8 \times .125 \\
&= \tfrac{8}{10} \times \tfrac{125}{1000} \\
&= \tfrac{1000}{10000} \\
&= .1
\end{aligned}
$$

$\therefore$ After multiplication, cut off as many places for decimals as are equal to the number of places of decimals in the multiplier and multiplicand.

105. To divide a Decimal by a Decimal.

$$
\begin{aligned}
&15{\cdot}345 \div 25{\cdot}57 \\
&= 15\tfrac{345}{1000} \div 25\tfrac{57}{100} \\
&= \tfrac{15345}{1000} \div \tfrac{2557}{100} \\
&= \tfrac{15345}{1000} \times \tfrac{100}{2557} \\
&= \tfrac{15345}{10} \times \tfrac{1}{2575} \\
&= \tfrac{15345}{2557} \times \tfrac{1}{10}.
\end{aligned}
$$

$\therefore$ from the factor $\frac{1}{10}$, it is seen that after division one decimal place should be set off in the quotient, because there is one more zero in the denominator of the dividend than in the divisor. Hence, in the quotient, set off for decimals as many places as those in the dividend exceed those in the divisor. If there are not as many in the dividend as in the divisor, annex zeros till the places are filled.

PER CENTAGE,

LESSON I.

106. **Per Cent.** means by the hundred, and is denoted by %, thus: 6 per cent. is denoted by 6%, 5 per cent. by 5%, &c.

107. Given the **Principal, Time** in **Years,** and **Rate;** to find the **Interest.**

1. What is the interest on 250 dollars for 3 years at 8%?

108. **Analysis:**

∵ the int. on 1 dol. in 1 year=8 cents*
" " 250 dols. " " =250×8=2000 cents,
and " " " " 3 years=3×2000=$60.00.

2. What is the interest on $800.50 for 5 years at 6%?
3. What is the interest on $450.75 for 2½ years at 8%?
4. Sold a horse for $60.00, for which I took a note payable in 3 years, with interest at 8%. What was due me?
5. Sold 1500 lbs. of cotton at 15 cents a pound, and

---

*Observe, this is eight cents.

took a note payable in 2 years at 8%. What was due me at the end of the time?

6. What is the interest on $25.00 for 8 years at 8%?

7. What is the interest on 750.33\frac{1}{3}$ for 2 years at 5$\frac{1}{2}$%?

8. Loaned $500.00, which was to be paid at the end of 3 years, with interest at 8%. If the payee fails, how much will I lose?

LESSON II.

109. Given **Principal, Time** in **Months,** and **Rate**; to find **Interest.**

1. What is the interest on $125.00 for 8 months, at 6%?

110. **Analysis:**

∵ the int. on 1 dollar in 12 months=6 cents,
" " " " " " 1 month=$\frac{1}{12}$ of 6 cents
=$\frac{1}{2}$ a cent,
" " " 125 dol. " " " =125×$\frac{1}{2}$=$\frac{125}{2}$ cents
and " " " " " " 8 months=8×$\frac{125}{2}$=$5.00.

2. What is the interest on $900.00 for 3 months at 8%?

3. What is the interest on $1200.60 for 5 months, at 7%?

4. Sold 40 bushels of oats at 40 cents a bushel, and took a note payable in 2 months, with interest at 8%. What should I receive?

5. What is the interest on $30.00 for 9 months at 8%?

6. Borrowed $50.00 with interest at 8%; and one month afterward sold the man from whom I borrowed the money 300 bushels of wheat at 60 cents a bushel. Show what would settle this business at the end of 4 months from the time the money was borrowed, both parties paying interest.

LESSON III.

111. Given **Principal, Time** in **Days,** and **Rate,** to find **Interest.**

1. What is the interest on $75.00 for 14 days, at 8%?

112. **Analysis.**

∵ the int. on 1 dol. in 360 days=8 cents,
" " " 1 day=$\frac{1}{360}$ of 8 cents
=$\frac{1}{45}$ of a cent,
" " 75 dols. in 1 day=$75\times\frac{1}{45}=\frac{5}{3}$ cents,
and " " " 14 days=$14\times\frac{5}{3}=\frac{70}{3}$
=$23\frac{1}{3}$ cents.

2. Excavated 6,000 cubic yards of earth at 18 cents per cubic yard, and took a note payable in 25 days, int. at 8%. What was due me?

3. What is the interest on $10.00 for 12 days, at 8%?

4. Loaned a friend $15.00 for 21 days at 6%. What is the interest?

5. Find the interest on $80.00 for 5 days, at 8%.

LESSON IV.

113. Given **Principal, Time** in **Years, Months** and **Days** and **Rate**, to find **Interest.**

114. Find interest for the **Years,** [108],
" " " **Months,** [110],
" " " **Days,** [112],
and add the results.

1. What is the interest on $656.75 for 2 years, 4 months and 10 days, at 8%.

2. What is the interest on $300.00 for 2 months and 8 days, at 6%?

3. What is the interest on $18.00 for 1 year, 1 month and 1 day, at 8%?

4. What is the interest on $10.00 for 2 years and 8 days, at 5%?

5. What is the interest on $15.00 for 5 years, 5 months and 12 days, at 8%?

6. What is the interest on $987.75 for 4 months and 10 days, at 8%?

7. What is the interest on $1200.00 for 7 years, at 6%?

8. What is the interest on $812.40 for 4 years, at 6%?

LESSON V.

115. Given **Principal, Time** and **Interest,** to find **Rate.**

1. Loaned $250.00 for 3 years, and received $45.00 interest. What is the rate?

116. **Analysis:**

∵ the int. on $250 for 3 years=45.00 cents,
" " " $1 " " $=\frac{1}{250}$ of 45.00$=\frac{90}{5}=$18 cents,
and " " " " " 1 year$=\frac{1}{3}$ of 18=6 cents, or 6%.*

2. Sold a horse for 50 dollars, after keeping him 20 days. If I paid 40 dollars for the horse, at what % had I invested my money?

Note.—Having gone to 1 dol. for 1 day, go to 1 dol. for 360 days.

3. Rented 10 acres in wheat for the third. The land yielded 25 bushels per acre. If the seed wheat and labor cost the renter 40 dollars, at what rate did the renter invest, wheat being worth 80 cents a bushel?

4. Paid 100 dollars for a lot on which I put a cottage at an outlay of 600 dollars. Rented the house for 75 dollars per year; what per cent did I get?

5. Put 500 dollars in a manufacturing company. In 4 months after taking the stock, it yielded 150 dollars. At what rate did I loan my money?

---

*What is derived from 1 dollar in 1 year is *rate per cent.*

6. Took a contract of grading a road, for 800 dollars. The grading cost me 500 dollars; and I did it in 3 months. What is the rate of investment?

7. Loaned 600 dollars for 2 years, 3 months, and 12 days, receiving 90 dollars interest. What was the rate per cent?

8. Bought 1000 dollars worth of goods, and, in 60 days, sold them for 1200 dollars. At what rate did I invest?

9. Loaned 30 dollars for 2 months, and received 30 cents. What was the rate of interest?

10. Bought $\frac{3}{4}$ of a mill valued at 1200 dollars, kept it 3 months, and sold it for 1200 dollars. What was my rate of interest?

LESSON VI.

117. Given **Principal, Interest,** and **Rate;** to find **Time.**

1. Loaned 50 dollars, for which, at 6%, I received 9 dollars interest. How long had it been on interest?

118. **Analysis:**

the interest on P* for 1 year, at 6%=3 dollars.
∴ ∵ to get 3 dols. int. on P=1 year,
" 1 dol " " $=\frac{1}{3}$ of $1=\frac{1}{3}$ of a year,
and " 9 dols " " $=9\times\frac{1}{3}=3$ years.

---

*P denotes principal.

Note.—I will give another way of analysing this problem.

119. 6% of *any* number$=\frac{6}{100}=\frac{3}{50}$ for 1 *year*.

∴ ∵ to get $\frac{3}{50}$ of P=1 year,
“ “ $\frac{1}{50}$ “ $=\frac{1}{3}$ of 1$=\frac{1}{3}$ of a year,
and “ “ $\frac{50}{50}$(=to P)$=50\times\frac{1}{3}$ of a year.
∴ ∵ “ “ 50 dols. int$=\frac{50}{3}$ years,
“ “ 1 dol. int$=\frac{1}{50}$ of $\frac{50}{3}=\frac{1}{3}$ of a year,
and “ “ 9 dols. int$=9\times\frac{1}{3}=3$ years.

(118) is the more simple.

1. Loaned 500 dollars, for which, at 8%, I received 140 dollars. How long was it on interest?

2. How long, at 8%, will it require 378 dollars to gain $78.50?

3. How long, at 6%, will it require 500 dollars to amount to 875 dollars?

4. Gave my note for 756 dollars, at 7%, and, at the end of a certain time paid $191.10. How long was it on interest?

5. How long will it require 450 dollars, at 7%, to amount to $620.10?

LESSON VII.

120. Given **Time, Interest,** and **Rate;** to find **Principal.**

1. What principal at interest for 2 years, at 6%, will produce 3 dollars interest?

121. **Analysis:**

6% of any Prin. at int. for 1 year$=\frac{6}{100}=\frac{3}{50}$ of P.

∴ ∵ int. of P for 1 year$=\frac{3}{50}$ of P.

" " 2 years$=2\times\frac{3}{50}=\frac{3}{25}$ of P.

∴ $\frac{3}{25}$ of P$=3$ dollars,

$\frac{1}{25}$ " $=\frac{1}{3}$ of $3=1$ dollar,

and $\frac{25}{25}$ " $=25\times1=25$ dollars.

2. After 3 years and 2 months I received $92.50. If R=6%, find P.

3. At the end of 5 yrs. 4 mo. and 24 da., I received $170.10. If money is worth 7%, what is P.

4. Received as standing rent for a little farm 125 dollars. If money is worth 8%, what principal at interest would yield as much?

5. Received $400.75 as my share of dividend on stock which yielded 9% annually. If I owned $\frac{1}{5}$ of the capital, find the capital.

6. If I received 10 dollars interest on a sum of money at interest for 1 year and 5 months, at 8%, what is the principal?

7. A company wishing to raise 800 dollars annually desires to know what sum at 8% must be put at interest.

8. Sold 500 bushels of wheat for which I took a note. At the end of 4½ years I received 135 dollars int. If money is worth 6%, what did I get a bushel for my wheat?

9. Took a ⅔ interest in a building company. The stock yielded 10%, and I received 250 dollars. What was the capital?

10. Loaned A 15 dollars to pay the interest on a sum of money which had been on interest 1 year and 7 months, at 8%. What sum was at interest?

LESSON VIII.

121. Given **Time, Rate,** and **Amount;** to find **Principal.**

1. At the end of 4 years I received 429 dollars as the amount of interest and principal. If money is worth 8% what is P?

122. **Analysis:**

8% of any prin. at int. for 1 year$=\frac{8}{100}=\frac{2}{25}$ of P.
∴ ∵ the int. of P. for 1* year$=\frac{2}{25}$ of P.
" " " 4 years$=4\times\frac{2}{25}=\frac{8}{25}$ of P.
∴ ∵ int. of P$=\frac{8}{25}$,
and P$=\frac{25}{25}$,
the amount$=\frac{33}{25}$.
∴ $\frac{33}{25}$ of P$=429$ dollars,
$\frac{1}{25}$ of P$=\frac{1}{33}$ of $429=13$ dols.
and $\frac{25}{25}$, or P$=25\times13=325$ dollars.

---

*If months are given, put 12 months for 1 year, and go to 1 month, then to the given number of months. If days, put 360, and proceed in the same way.

2. What prin. at 7% will amount to $1941.04 in 11 months?

3. What prin. at 8% for 3¼ years will amount to $1248.50?

4. What prin. at 8% for 55 days will amount to $2,094,498?

5. My agent collected a note amounting to 60 dollars, which had been on interest one year, at 8%. What was the principal?

6. Ordered 4 wheat drills for which I gave my note, at 8%. After 4 years and 15 days I paid $661.66⅔. What was the price of each drill?

7. Bought flour at a certain price per barrel. After 90 days I sold the flour at 7 dollars per barrel. If money is worth 8%, what did this flour cost me per barrel?

8. At a store I was told that I could have oats at 40 cents a bushel cash, or 41 cents and wait 2 months for the money. If money is worth 8%, which is the more advantagous, borrow the money and pay 40 cents, or wait 2 months and pay 41 cents?

9. If sugar is sold at 9 cents a pound, on 60 days time, money being worth 8%, what did it cost a pound?

10. Sold a horse for 70 dollars which was collectable after 4 months. If by the above price I loaned my money at 8%, what did the horse cost me?

123. Frequently in business payments are made on notes at *different* times. These different payments are called **Partial Payments.** These partial payments arise by virtue of the fact that it suits the convenience of the borrower to pay his note in this way.

124. A **Fundamental** principle is *that the interest shall be paid first.* Hence,

125. If at any time the **Payment** is less than the **Interest, Nothing** has been paid on the **Principal**.

$350.00.

1. One day after date, I promise to pay John Smith three hundred and fifty dollars, for value received, int. at 8%.

JOHN JONES, [Seal.]

Aug. 15, 1890.

On this note were the following payments:

Nov. 22, 1890, - - - - - - -	$50.00
June 10, 1891, - - - - - - - -	10.00
Oct. 1, 1891, - - - - - - -	75.00

Dec. 25, 1891, what was due?

126. **Analysis and Arrangement of Work:**

Time from date of note to first paym't,	3 mo.	7 da.,	$50.00
" " first paym't to second	" 6	" 18 "	$10.00
" " second " " third	" 3	" 21 "	75.00
" " last " " settlement,	2	" 24 "	X.

Int. on $350.00 for 3 mo. 7 da. @ 8%=$7.54,

∵ the payment=50.00,

and " interest=7.54,

that paid on the note=42.46.

∴ the new prin.=350.00—42.46

=$307.54.

Int. on $307.54 for 6 mo. 18 da. @ 8%=13.53

∵ the interest=13.53,

and " payment=10.00,

unpaid interest=$3.53.

∴ by (125) nothing has been paid on the *last* prin. $307.54.

∴ int. on 307.54 for 3 mo. 21 da., @ 8%=$7.53.

∵ the int.=7.55,

and the unpaid int.=3.53,

total interest=11.08.

∵ the payment=75.00

and total interest=11.08,

that paid on note=63.12

∴ the new prin.=307.54—63.14

=$244.42.

Int. on $244.42 for 2 mo. 24 da. @ 8%=$4.47.

∴ that due on Dec. 25, 1891,=244.42+4.47

=$248.89.

2. One day after date, I promise to pay $500.00 to O. Harris, (five hundred dollars,) for value received, with interest at 8%.

J. K. WEST, [Seal.]

Sept. 1, 1890.

On this note were the following endorsements:

Dec. 15, 1890,	$150.00
May 5, 1891,	$200.00
Oct. 7, 1891,	$ 50.00

Jan. 1, 1892, what was due?

This subject, though important, is not difficult. If the teacher thinks it necessary, he can illustrate it further by making several more examples. Before taking leave of this subject, the author wishes to state that there are other views of it. As they are based on greed, not on mathematics, he does not wish to waste time with them.

When at the end of a year the interest on any principal has been computed and added to the principal, and this sum (=to amount) is placed at interest for another year, the interest again being computed and added, and this amount placed at interest again, &c., the interest at the end of a given time is called **Compound Interest.**

COMPOUND INTEREST.

LESSON I.

1. What is the compound interest of $400.00 for 7 years, at 8%?

128. **Analysis:**

Int. on $400 for 1 yr. @ 8%=$32.00.
∴ amount=$432.00.
Int. on $432 for 1 yr. at 8%=$34.56.
∴ amount=$466.56.
Int. on $446.56 for 1 yr. at 8%=$37.4248.
∴ amount=$503.88.
Int. on $503.88 for 1 yr. at 8%=$40.3004.
∴ amount=$544.18.
∵ the amount for 4 years=$544.18.
and " principal=$400.00.
" compound interest=$544.18—400.00.
=$144.18.

2. What is the compound interest on $250,875 for 3 years and 2 months,* af 8%?

3. What is the compound int. on $75.50 for 5 years, at 6%?

---

*Having proceeded for the years as above, calculate int. on all for 2 months.

4. What is the compound int. on $1000.00 for 10 years at 10%?

5. What is the compound int. on $175.00 for 2 years and 15 days, at 6%?

This is deemed sufficient on this subject. Many other interesting and useful problems of compound interest there are; but the author did not set out with the intention of using principles based upon Algebra. To Algebra, therefore, belongs many problems in compound interest and annuities.

TRUE DISCOUNT.

129. In business it often happens that a note is made payable at a future time without interest. If the holder needs ready money, he must sell the note. The buyer wishes interest on his money. He, therefore, pays **Such** a sum as placed at a given interest for a given time will amount to the **Face** of the note.

130. This **Sum** (paid now) is called **Present Worth.**

131. The **Difference** between the **Present Worth** and the **Face** of the note is called **Discount.**

LESSON I.

1. What is the present worth of a note of $450.00 payable in 4 years, at 8%?

132. **Analysis:**

8% of any prin.$=\frac{8}{100}=\frac{2}{25}$ for 1 year.
int. on P. for 1 yr.$=\frac{2}{25}$ of P,
"   "   4 yrs.$=4\times\frac{2}{25}$ of P$=\frac{8}{25}$ of P=Discount.
∴ $\frac{25}{25}=$P,
and $\frac{8}{25}=$int.,
$\frac{25}{25}+\frac{8}{25}=\frac{33}{25}=$amount.
∴ $\frac{33}{25}=450.00$ dol.,
$\frac{1}{25}=\frac{1}{33}$ of $450.00=\frac{450}{33}$,
and $\frac{25}{25}=25\times\frac{450}{33}=340.90\frac{10}{11}$ dol.

2. What is the discount on \$200.00 for 1 year and 3 months, at 8%?

133. **Analysis.**

8% of P. for 1 year $=\frac{8}{100}=\frac{2}{25}$ of P.
∵ int. on P. for 12 mos. $=\frac{2}{25}$ of P,
" " 1 mo. $=\frac{1}{12}$ of $\frac{2}{25}=\frac{1}{150}$ of P,
and " " 15 mos. $=15\times\frac{1}{150}=\frac{1}{10}$ of P=discount.
∴ ∵ $\frac{10}{10}=$ P,
and $\frac{1}{10}=$ D,
$\frac{10}{10}+\frac{1}{10}=\frac{11}{10}=$ amount,
∴ $\frac{11}{10}=$ 200 dol.,
$\frac{1}{10}=\frac{1}{11}$ of $200=\frac{200}{11}$,
$=18.18\frac{2}{11}=$ D.

3. Was offered a horse for 75 dollars cash, or 80 dollars payable in 8 months. If money is worth 6%, which is the better bargain, and by how much?

4. What is the present worth of \$500.00 due in 5 years, at 6%?

5. Bought corn at a certain price; and, after 60 days, finding the market close, sold it at 40 cents a bushel. If the interest on my money was all that I desired, and money was worth 8%, how much did the corn cost a bushel?

6. Asked 8 cents a lb. for sugar at the end of 45 days. If money is worth 6%, what did the sugar cost a lb.?

7. What is the discount on a note of \$375.00 for 3 yrs. 2 mo. and 11 days, at 8%?

8. What should I pay for a note of $75.00, payable in 1 year and 5 months, if money is worth 8%?

9. Bought 40 bushels of corn for which I was to pay $25.00 in 5 months. If money is worth 8%, what did it cost me per bushel down?

10. Gave my note for $8,000, payable in 1 year. If money is worth 8%, at what price should it be discounted?

BANK DISCOUNT.

135. A **Bank** is an institution whose chief object is to loan money.

136. If a given sum, for a given time, at a given per cent., is borrowed at a bank, the banker will calculate the interest on the *given sum*, for the time, and at the given rate, and, deducting this interest from the sum, or note, pay you the **Difference.**

137. This interest, calculated for **G. T.**, at **G. R.**, is called **Bank Discount.**

LESSON I.

1. Wish to borrow \$50.00 from a bank, for 90 days. If discounted at the bank, what is the discount, and what do I receive, money being worth 8%?

138. **Analysis:**

$\because$ 1 dol. in 360 days $= 8¢$ interest,
1 " " 1 day $= \frac{1}{360}$ of 8 cents $= \frac{1}{45}$,
50 dollars " " $= 50 \times \frac{1}{45} = \frac{10}{9}$ cents,
and " " " $93^* = 93 \times \frac{10}{9} = \frac{310}{3}$
$= \$1.03\frac{1}{3}$.
$\therefore$ discount $= \$1.03\frac{1}{3}$,
and what is received $= 50.00 - 1.03\frac{1}{3}$
$= \$48.96\frac{2}{3}$.

---

*Banks allow 3 days beyond the specified time for money to reach them. These days are called days of *grace*.

2. Let us take the same problem, and see how much money you would receive from the bank by *true* discount.

Analysis:

8% of any P for 1 yr$=\frac{8}{100}=\frac{2}{25}$ of P.
$\therefore \because$ int. on P for 360 da.$=\frac{2}{25}$ ot P,
" " 1 da.$=\frac{1}{360}$ of $\frac{2}{25}=\frac{1}{4500}$ of P,
and " " 93 ds.$=93\times\frac{1}{4500}=\frac{31}{1500}$ of P.
$\therefore \because$ $\frac{1500}{1500}=$P,
and $\frac{31}{1500}=$int., or [Discount true],
$\frac{1500}{1500}+\frac{31}{1500}=$amount.
$\therefore$ $\frac{1531}{1500}=\$50.00$,
$\frac{1}{1500}=\frac{1}{1531}$ of $50.00=\frac{50}{1531}$,
and $\frac{1500}{1500}=1500\times\frac{50}{1531}$
$=\frac{75000}{1531}$
$=\$48,987$.*

3. Wish to borrow $500.00 from the bank for 1 year. If money is worth 8%, how much will I receive from the bank?

4. A note of $80.00 payable in 4 months is discounted at the bank at 8%. What is the discount?

5. What is the amount which you receive from a bank, if you apply for $800.00, payable in 2 years, at 8%?

6. What is the difference between the true and the bank discount in the above example?

7. Took a note from A for $500.00, payable in 4

---

*Which is just and right to the borrower?

months, at 8%. If discounted at the bank, how much do I receive?

8. What I receive in the above is the present worth of how much, for the time and rate given?

LESSON II.

139. If in case of the fifth problem above, you had needed all of the $800.00 (and this is frequently the case) and, therefore, you had drawn $800.00 out of the bank, it would have been necessary to give your note, so as to secure the banker his interest in accordance with (136).

140. This is called making a note at bank payable at a **Future** time.

141. **Analysis:**

8% of P for 1 year $=\frac{8}{100}=\frac{2}{25}$ of P

$\therefore\ \because$ int. on P for 360 days $=\frac{2}{25}$ of P,

" " 1 day $=\frac{1}{360}$ of $\frac{2}{25}=\frac{1}{4500}$ of P,

and " " 723 days $=723\times\frac{1}{4200}=\frac{47}{500}$ of P.

$\therefore\ \because$ $\frac{500}{500}=$ P,

and $\frac{47}{500}=$ int.,

what you receive $=\frac{500}{500}-\frac{47}{500}$

$=\frac{453}{500}$.

$\therefore$ $\frac{453}{500}=\$800.00$,

$\frac{1}{500}=\frac{1}{453}$ of $\$800.00=\frac{800}{453}$,

and $\frac{500}{500}=500\times\frac{800}{453}=\frac{400000}{453}$

$=\$880.002$.

2. What must be the face of a note at bank, payable in 60 days, at 8%?

3. Wishing to borrow $600.00 at bank, for what sum must I draw my note? The time and rate are 2 months and 6%.

4. Gave my note to an individual for $600.00 payable in 2 months. What price should a man pay for this note cash, money being worth 6%?

Note.—See (132).

5. What should be the face of a note at bank, payable in 30 days, at 8%?

6. Desirous of borrowing $1200.00 from the bank, for what sum must I draw my note payable in 3 months, at 8%?

7. What is the difference between true and bank discount on $1000.00 payable in 4 months, at 8%?

REVIEW.

1. Placed $500.00 out at compound interest, at 8%, for 3 years. What is its present worth at 6%?

2. Was asked $10.00 an acre for land 1 year hence. At what price cash could it be bought, if money is worth 8%?

3. What could you get per acre, if you sold the above note to a bank?

4. Gave my note to A for $750.00, payable in 5 months

at 8%. A unexpectedly became in need of ready money, and had the note discounted at the bank. How much did he get.

5. If he took out the face of the above note, for what should he draw his note?

6. Rented my house and lot for $80.00 payable at the end of the year. What is it worth cash?

7. What per cent is this cash value on the cost of the house at $800.00?

8. Sold $690.00 of tobacco which during the year cost me $175.00. What rate of interest did I get?

9. If I sold the above 2 months on time, what did I get down, money being worth 8%?

142. Up to this point we have been considering per centage connected with time. But as various transactions of business are carried on without considering time an Element, we will notice per centage irrespective of time.

143. Basal:

1% of any number $=\frac{1}{100}$ of it,
2 " " " $=\frac{2}{100}=\frac{1}{50}$, [always reduce to lowest terms.]
3 " " " $=\frac{3}{100}$,
4 " " " $=\frac{4}{100}=\frac{1}{25}$,
5 " " " $=\frac{5}{100}=\frac{1}{20}$,
6 " " " $=\frac{6}{100}=\frac{3}{50}$,
&c.

Note.—No time is connected with this.

PROFIT AND LOSS.

LESSON I.

1. Sold a knife for 18 cents by which I gained 20%. What per cent would I have gained had I sold it for 19 cents?

143. **Analysis:**

20% of a number $=\frac{20}{100}=\frac{1}{5}$ of it.
∴ $\frac{5}{5}$=cost,
$\frac{1}{5}$=gain,
and $\frac{5}{5}+\frac{1}{5}=\frac{6}{5}$=sale.
∴, ∵ $\frac{6}{5}$=18 cents,
$\frac{1}{5}=\frac{1}{6}$ of 18=3 cents,
and $\frac{5}{5}=5\times3=15$ cents,
∵ 19¢=sale, last,
and 15¢=cost,
19—15=4 cents gain.
∵ a gain of cost, or 15¢=100%,
" " 1¢ $=\frac{1}{15}$ of 100% $=\frac{20}{3}$%,
and " " 4¢ $=4\times\frac{20}{3}=\frac{80}{3}$%,
$=26\frac{2}{3}$%.

2. Bought sugar at 6 cents a pound, and sold it at 8 cents a pound. What % did I make on my investment?

3. Bought 60 gallons of molasses at 40 cents a gallon.

20% having leaked out, at what price per gal. must I sell the remainder to make 25% on the cost?

Analysis:

∵ the cost of 1 gal. $=40$¢,
" " 60 gals. $=60\times 40=\$24.00$.
Now 20% of a number $=\frac{20}{100}=\frac{1}{5}$ of it.
∴ what remained $=\frac{5}{5}-\frac{1}{5}=\frac{4}{5}$.
But $\frac{5}{5}=60$ gallons,
$\frac{1}{5}=\frac{1}{5}$ of $60=12$ gals.,
and $\frac{4}{5}=4\times 12=48$ gallons.
Again, 25% of a number $=\frac{25}{100}=\frac{1}{4}$ of it.
∴ ∵ the cost $=\frac{4}{4}$,
and " gain $=\frac{1}{4}$,
" sale $=\frac{4}{4}+\frac{1}{4}=\frac{5}{4}$.
But ∵ $\frac{4}{4}=\$24.00$,
$\frac{1}{4}=\frac{1}{4}$ of $24.00=6.00$,
and $\frac{5}{4}=5\times 6.00=\$30.00$.
∴ ∵ 48 gallons $=\$30.00$,
1 gallon $=\frac{1}{48}$ of $30.00=\frac{3000}{48}$
$=\$0.625$.

4. Bought calico at 8¢ a yard. For what must I sell it per yd. to make 25%?

5. Sold shoes at $24.00 a doz., by which I lost 20%. What did they cost me a pair?

6. Asked $5.00 a bbl. for flour, which was an advance of 25% on the cost. Would I have gained, had I sold it for $4.50? If so, what per cent. would I have made?

7. Sold eggs at 15 cents a doz., by which I gained 25%. What did the eggs cost me a doz.?

8. If I paid for the above eggs in goods by which I made 33⅓%, what per cent did I make on the original investment?

9. Invested $800.00 in cotton on which I made 20%. What gain did I realize?

10. Paid $600.00 for a tract of land, and sold it at a loss of 33⅓%. What did I get for the land?

11. Bought 400 feet of boards at $11.00 per C. and paid 50¢ for draying them home. In drying them I lost 25% by fire. At what price per hundred must I sell the remainder to make 20% on cost?

12. Bought 100 hides at 80¢ a hide. Lost 25% of hides. At what price per hide must I sell the remainder, in order to gain 12½% on cost?

Note.—12½% of a number $=\frac{\frac{25}{2}}{100}=\frac{25}{200}=\frac{1}{8}$.

13. Sold a pair of shoes for $3.00 by which I lost 25%. What did they cost me?

LESSON II.

1. Sold a quantity of wheat so that $\frac{3}{4}$ of the selling price equals the cost. What per ct. did I make?

145. **Analysis:**

$\because$ $\frac{3}{4}$ of sale=cost,
$\frac{1}{4}$ of sale=$\frac{1}{3}$ of the cost,
and $\frac{4}{4}$, or whole sale=$4 \times \frac{1}{3}=\frac{4}{3}$ of the cost.
$\therefore$ $\frac{4}{3}$=sale,
and $\frac{3}{3}$=cost,
$\frac{4}{3}-\frac{3}{3}=\frac{1}{3}$=gain.
But $\because$ to gain $\frac{3}{3}$=100%,
" " $\frac{1}{3}=\frac{1}{3}$ of 100%=$33\frac{1}{3}$%.

2. Bought an article in such a way that, when I sold it, $\frac{7}{8}$ of the cost was equal to the selling price. What per ct. did I lose?

3. Sold cotton so that $\frac{9}{10}$ of the sale equals the cost. What per cent. did I make?

4. A farmer sells corn at 75 cents per bushel, and gains 25%. What would he have gained or lost, if he had sold it at 55 cts. per bushel? What per cent. would he have gained or lost?

5. Sold calico so that $\frac{4}{5}$ of the selling price equals the cost. What per cent. did I make?

6. Sold $400.00 worth of goods at a loss of $33\frac{1}{3}$ per cent. What did they cost me?

COMMISSION.

146. Commission is a per cent. allowed an agent for **Buying** and **Selling** goods.

LESSON I.

1. My agent collected $60.00 for me, for which he charged me 20% for collecting. How much did he remit and how much did he keep?

147. **Analysis:**

20% of a number $=\frac{20}{100}=\frac{1}{5}$ of it.

$\therefore\ \because$ agent's share $=\frac{1}{5}$,

my " $=\frac{5}{5}-\frac{1}{5}=\frac{4}{5}$.

But $\frac{5}{5}=\$60.00$,

$\frac{1}{5}=\frac{1}{5}$ of $\$60.00=\$12.00$ [agt's share],

$\frac{4}{5}=4\times\$12.00=\$48.00$.

2. Again the agent collected $60.00, and I paid him for the same 20% of *all* he remitted me. How much did I get, and what did the agent retain?

148. **Analysis.**

20% of a number$=\frac{20}{100}=\frac{1}{5}$ of it.
my part$=\frac{5}{5}$,
agent's " $=\frac{1}{5}$,
and both parts$=\frac{5}{5}+\frac{1}{5}=\frac{6}{5}$.
But $\frac{6}{5}=\$60.00$,
$\frac{1}{5}=\frac{1}{6}$ of $60.00=\$10,00$,
and $\frac{5}{8}=5\times 10=\$50.00$

3. Sent my agent at Charlotte $1500.00 with which to buy cotton, allowing a commission of 25% on what he spent for me. What is his commission, and how many lbs. can he buy for me at 10¢ a pound?

4. At what must I sell the cotton per lb., to gain 20% on the outlay?

5. Hats which cost me $1.00 apiece, I sold at a discount of 50% on my asking price. What did I ask a piece?

6. Directed my agent at Richmond to by me $300.00 worth of ham. If he charges 2% for all the money sent him, how much ham can he buy at 9¢ a pound?

7. Sent my agent $1200.00 with which to buy cotton, allowing him 2% on all he spent for me. How much did he spend for me?

8. If in trading, I gained 20% on the above outlay, for what would a note of the same amount be discounted at the Trinity Bank?

STOCKS.

149. Stock is a general name for capital employed in **Trade, Manufactures, Railroads, Banking, Telegraphs, &c.**

150. Money lent to the government is called **stock.**

151. Capital invested by companies is called **stock**.

152. The capital is generally divided into **shares** of 100 dollars each.

153. The face of a share is its **Par Value.**

154. When the stock sells for less than its face, it is **Below** par.

155. When the stock sells for more than its face, it is at a **Premium.**

LESSON I.

1. A man buys 12 shares of R. & D. R. R. stock* at a premium of 15%. What does it cost him?

156. **Analysis:**

∵ the value of 1 share=100 dollars,
" " " 12 shares=12×100=$1200 (par val).
Now 15% of a number=$\frac{15}{100}=\frac{3}{20}$ of it.
∴ ∵ $\frac{20}{20}$=par val.,
and $\frac{3}{20}$=premium,
$\frac{20}{20}+\frac{3}{20}=\frac{23}{20}$=cost.
But $\frac{20}{20}$=$1200.00,
$\frac{1}{20}=\frac{1}{20}$ of 1200.00=60.00,
and $\frac{23}{20}=23\times60.00$=$1380.00.

---

*When not specified, the share is 100.

2. Purchased 10 shares of stock in the Randleman Mills at a discount of 8%. What did they cost me?

3. If the above stock declared a dividend of 15%, what was my income?

4. At what per cent. had I loaned my money invested?

5. Employed my agent to buy 25 shares in the C. F. & Y. V. R. R., which was at a premium of 20%. What do they cost me, allowing the agent a commission of 2%?

Note.—It must be borne in mind that the commission is calculated on the par value, and *not on the market value.*

Analyze thus:

20% of a number $=\frac{20}{100}$ (premium),
2% " " $=\frac{2}{100}$ (commission).
∴ $\frac{20}{100}+\frac{2}{100}=\frac{22}{100}=\frac{11}{50}$ above par.
∴ ∵ $\frac{50}{50}=$ par val.,
and $\frac{11}{50}=$ prem. and com.,
$\frac{50}{50}+\frac{11}{50}=\frac{61}{50}$ cost.
But $\frac{50}{50}=2500.00$,
$\frac{1}{50}=\frac{1}{50}$ of $2500.00=50.00$,
and $\frac{61}{50}=61\times50.00=\$3050.00$.

6. If a cotton mill declares a dividend of 8%, what sum will a stockholder who has 5 shares receive?

7. Sold through my broker 20 shares of stock in the Hawking Valley R. R., at a discount of 3%. If broker charges 2%, what was lost on the stock?

8. If the Trinity Mills pay 25% dividend on the stock, what is the income of him who owns 5 shares?

9. Bought through my agent 25 shares in R. & D. R. R. at a premium of 25%, and sold through him the same at a discount of 20%, paying brokerage of $\frac{1}{2}$% both ways. How much did I gain?

10. If a dividend of 8% is declared on the R. & D. R. R. stock, how many shares does an individual who receives $600.00 own?

LESSON II.

1. How much stock can be bought in the H. R. & A. R. R. with $57,000.00, if the stock is at a discount of 5%?

157. **Analysis:**

5% of a number $=\frac{5}{100}=\frac{1}{20}$ of it.
∴ $\frac{20}{20}=1$ dol. of the stock,
and $\frac{1}{20}=$loss in 1 dol.
$\frac{20}{20}-\frac{1}{20}=\frac{19}{20}=$cost of 1 dol.
∴ $\frac{19}{20}$ of 1 dol. of my money=1 dol. of the stock,
$\frac{1}{20}$ " " " " $=\frac{1}{19}$ of 1 doll.$=\frac{1}{19}$ of a dol. of stock,
$\frac{20}{20}$ or 1 dol. of my money$=20\times\frac{1}{19}=\frac{20}{19}$ " " "
and 57,000 dollars " " $=57{,}000\times\frac{20}{19}=60{,}000$ dollars of stock.

2. How many shares of $50.00 each can I buy in the C. F. & Y. V. R. R. with $7500.00 at a premium of 9%?

3. Invested $1000.00 in the High Point Sash and Blind Co. How much stock can I buy, if the stock is at a premium of 6%?

4. If the stock above declares a dividend of 10%, at what rate did I invest my money?

5. How much stock can I buy with $1500.00, at a discount of 5%?

LESSON III.

1. At what price must I buy N. C. 6's that I may realize 8% on my money?

158. **Analysis:**

∵ to get 6¢ int. @ 6%=$1.00 of money paid out,
“ “ “ “ 1%=6×100=$6.00 “ “ ,
and “ “ “ “ 8%=$\frac{1}{8}$ of 6.00=75¢ “ “ .

∴ the stock will cost 75¢ in the dollar.

2. At what price must I buy N. C. 5's that I may loan my money at 6%?

3. At what price above par must I buy New York 6's that they may yield 5%?

4. At what price below par must I buy a note bearing 6% that I may realize 8%?

5. If the above note was $100.00, what was the total per cent. realized?

REVIEW.

1. Bought land at $10.00 an acre. At what price per acre must I sell it in order to abate 10% on the purchase money, and make 20% on total cost?

2. Bought hats at $1.00 apiece. For what must I sell them apiece, if I sell them so as to lose 25% on my asking price?

3. Bought sugar at 6¢ a pound. At what price per pound must I sell it that I may gain 33⅓%?

4. Having directed my agent to buy cotton for me, how many pounds at 11¢ a pound can he buy with $1200, if he charges 2% on all he spends for me?

5. Which is the better investment: 25 shares in R. & D. Co. at 20% premium, yielding a dividend of 9%; or 25 shares in C. F. & Y. V. at a discount of 20%, yielding a dividend of 6%?

6. How much is gained on an investment of $1500.00, at an advance of 25%?

How much stock can I buy with $3000.00, at a discount of 8%?

8. If gold is worth $1.50, at what price must U. S. bonds at $6%, sell that I may realize 8% on my money?

9. What premium must be paid for insuring goods valued at $8760, at 2¾% per annum?

10. A man directs his broker to purchase 72 shares of stock in a mill at an advance of 2%, and afterwards sells

the same at a premium of 5%; what is the entire profit if the broker charges ¼% for buying and selling the same stock?

11. Sold eggs at 15¢ a dozen, by which I made 25%. If I had paid for the eggs in goods by which I had made 33⅓%, what did the eggs cost me per doz.?

12. If 376 dollars have been realized as a profit of 8% on an investment, what was the investment?

13. Bought a book for 80¢, and sold for 90¢; what per cent did I make?

CO-PARTNERSHIP.

250. When a number of men have put money into a business, for the purpose of sharing the gains and losses in proportion to the capital of each individual, it is called **Co-partnership.**

160. Different sums may be employed for the **Same Time.** Here **Time** is not an element of consideration.

161. Different sums may be employed for **Different Times.** Here **Time** is an element of consideration?

LESSON I.

1. A. put in trade 40 bushels of oats at 50¢ a bushel, B. put in 30 gallons of molasses at 60¢ a gal., and C. put 12 bushels of wheat at $1.00 per bus. They gained $25.00; what was each man's share of the gain?

162. **Analysis:**

40 bushels of oats @ 50¢=20.00,
30 gals. molasses @ 60¢=18.00,
and 12 bus. wheat @ $1.00=12.00
∴ amount of capital=$50.00.
∴ ∵ 50 dollars=$25.00 gain,
1 dollar =$\frac{1}{50}$ of 25.00=$\frac{1}{2}$ dol. gain,
20 dollars=20×$\frac{1}{2}$=$10.00, or A.'s,
18 " =18×$\frac{1}{2}$=$9.00, or B.'s,
and 12 " =12×$\frac{1}{2}$=$6.00, or C.'s.

2. A, B, and C worked a tabacco farm in the following way: A. furnished 10 acres of land at $2.50 an acre, B. worked 19 days at 75¢ a day, and C. worked 21 days at 75¢ a day. The tobacco brought $300.00. What was each man's share of the money?

3. Smith, Jones, and English go to market. Smith carries 60 chickens @ 15¢ apiece, Jones 75 lbs. butter @ 20¢ a pound, and English 500 lbs. ham @ 12$\frac{1}{2}$¢ a pound. Their expenses were jointly $9.00. How much did each pay?

4. A. owed B. $200.00, C. $300.00, and D. $75.00. How much should each get, if A. has only $90.00?

LESSON II.

1. A. put $300.00 for 4 months, B. put $200.00 for 3 months, and C. put $500.00 for 5 months. They gained $150.00. How much was each one's share of the gain?

163. **Analysis:**

300 dols. in trade 4 mos = 1 dol. in $300 \times 4 = 1200$ mos.,
200 " " 3 " = " " $200 \times 3 = 600$ "
500 " " 5 " = " " $500 \times 5 = 2500$ "
∴ 1 dol. was in trade $1200 + 600 + 2500 = 4300$ "
∴ ∵ the gain of
1 dol. for 4300 mos. = $150,
do 1 mo. $= \frac{1}{4300}$ of $150 = \frac{3}{86}$ dol.,
do 1200 mos. $= 1200 \times \frac{3}{86} = \frac{3600}{86} = 41\frac{7}{43}$ dols. (A's)
do 600 mos. $= 600 \times \frac{3}{86} = \frac{1800}{86} = 20\frac{20}{43}$ " (B's)
and do 2500 mos. $= 2500 \times \frac{3}{86} = \frac{7500}{86} = 87\frac{9}{43}$ " (C's)

2. A. pastured 8 cows for 15 days, B. pastured 16 calves for 8 days, and C. pastured 5 colts for 6 days. The pasture cost them 20 dollars. How much should each pay?

3. A. promised to pay 40 dollars in 2 months, 60 dols. in 3 mos., and 70 dols. in 4 mos. When may it all be paid?

4. If the above was paid at one time, in 7 months from

the beginning, for how long should the party to whom the money was due charge interest?

5. A. went into business with a capital of 1000 dollars, 4 months afterward B. entered with a capital of 1200 dollars. 15 months after A. began business, he withdrew 500 dollars; and, at the end of 18 months from the beginning, they had gained 800 dollars. What was each one's share of the gain?

6. A, B, and C. take an interest in a mill. A. puts in 500 dollars for 6 months, B. a sum not named for 12 mos, and C. 350 dols. for a time not named. When the accounts were settled, A. received 700 dols. for his stock and profits; B. 1200 dols for his; and C. 600 dols. for his. What were B.'s stock and C.'s time?

7. A, B, and C. form a copartnership. A.'s stock is in trade 4 months, and he claims $\frac{1}{12}$ of the gain; B's stock, is in trade 8 mos.; and C. put in 600 dollars for 4 mos. and claims $\frac{1}{2}$ the profits. How much did A. and B. put in trade?

DEC 5 1898